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A faster algorithm to recognize even-hole-free graphs ☆

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ABSTRACT

We study the problem of determining whether an n-node graph G contains an *even hole*, i.e., an induced simple cycle consisting of an even number of nodes. Conforti, Cornuéjols, Kapoor, and Vušković gave the first polynomial-time algorithm for the problem, which runs in $O(n^{40})$ time. Later, Chudnovsky, Kawarabayashi, and Seymour reduced the running time to $O(n^{31})$. The best previously known algorithm for the problem, due to da Silva and Vušković, runs in $O(n^{19})$ time. In this paper, we solve the problem in $O(n^{11})$ time via a decomposition-based algorithm that relies on the decomposition theorem of da Silva and Vušković. Moreover, if G contains

☆ The current version slightly improves upon the preliminary version [4] appeared in SODA 2012: (a) The time complexity for recognizing even-hole-free n-node m-edge graphs G is reduced from $O(m^2n^7)$ to $O(m^3n^5)$, which is an improvement if $m = o(n^2)$; and (b) if G contains even holes, then the current version shows how to output an even hole of G also in $O(m^3n^5)$ time.

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even holes, then our algorithm also outputs an even hole of G in $O(n^{11})$ time.

1. Introduction

For any graphs G and F, we say that G *contains* F if F is isomorphic to an induced subgraph of G. If G does not contain F, then G is F*-free*. For any family $\mathbb{F}$ of graphs, G is $\mathbb{F}$*-free* if G is F-free for each graph F in $\mathbb{F}$. A *hole* is an induced simple cycle consisting of at least four nodes. A hole is *even* (respectively, *odd*) if it consists of an even (respectively, odd) number of nodes. Even-hole-free graphs have been extensively studied in the literature (see, e.g., [1,13–15,20,21,30,38]). See Vušković [43] for a recent survey. This paper studies the problem of determining whether a graph contains even holes. Let n be the number of nodes of the input graph. Conforti, Cornuéjols, Kapoor, and Vušković [12,16] gave the first polynomial-time algorithm for the problem, which runs in $O(n^{40})$ time [7]. Later, Chudnovsky, Kawarabayashi, and Seymour [7] reduced the running time to $O(n^{31})$. Chudnovsky et al. [7] also observed that the running time can be further reduced to $O(n^{15})$ as long as prisms can be detected efficiently, but Maffray and Trotignon [31] showed that detecting prisms is NP-hard. The best previously known algorithm for the problem, due to da Silva and Vušković [21], runs in $O(n^{19})$ time. We solve the problem in $O(n^{11})$ time, as stated in the following theorem.

Theorem 1.1. *It takes $O(m^3n^5)$ time to determine whether an n-node m-edge connected graph contains even holes.*

Technical overview. The $O(n^{40})$-time algorithm of Conforti et al. [16] is based on their decomposition theorem [15] stating that a connected even-hole-free graph either (i) is an extended clique tree, or (ii) contains non-path 2-joins or k-star-cutsets with $k \in \{1, 2, 3\}$. The main body of their algorithm recursively decomposes the input graph G into a list $\mathbb{L}$ of a polynomial number of smaller or simpler graphs using non-path 2-joins or k-star-cutsets with $k \in \{1, 2, 3\}$ until each graph in $\mathbb{L}$ does not contain any of the mentioned cutsets. Since even holes in extended clique trees can be detected in polynomial time, it suffices for their algorithm to ensure the even-hole-preserving condition of $\mathbb{L}$: G is even-hole-free if and only if all graphs in $\mathbb{L}$ are even-hole-free. To ensure the condition of $\mathbb{L}$, their algorithm requires a cleaning process to either detect an even hole in G or remove bad structures from G before obtaining $\mathbb{L}$ from G. The $O(n^{31})$-time algorithm of Chudnovsky et al. [7], which is not based upon any decomposition theorem but still requires the cleaning process, looks for even holes directly. (The algorithms of Chudnovsky et al. [6] for recognizing perfect graphs are also of this type of non-decomposition-based algorithms.) The $O(n^{19})$-time algorithm of da Silva and Vušković [21], adopting the