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ABSTRACT

This article attempts to survey Alan Dow's contributions to General and Set-theoretic Topology.

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Alan Dow celebrated his 60th birthday on the 5th of December 2014 by flying to Ithaca, NY, to undergo a conference in honour of that birthday. This article is an attempt to present an overview of Alan's many contributions to General and Set-theoretic Topology.

1. Elementarity

One of the many gifts from Alan to Set-theoretic Topology is the use of elementarity. For a while this was even known as "Dow's method of elementary submodels". But Alan would be, was, and still is the first to protest that the Löwenheim–Skolem theorem predates him by a few decades.

We have for the longest time been familiar with recursive constructions where often beforehand a sequence of situations/sets is set up and during the construction witnesses to bad things will be eliminated or witnesses to good things will be embraced. In the end we consider such a situation and realize that it was basically dealt with during the construction. A very good example is the Pol–Shapirovskiĭ proof of Arhangel'skiĭ's theorem on the cardinality of compact first-countable spaces.

What set theorists realized was that one can reduce the length of such proofs considerably by an application of the Löwenheim–Skolem theorem to a model of 'enough set theory': its proof is the ultimate closing-off argument where one deals with *all possible* situations in one go (even ones that will never occur in your problem at hand). But, and this is where this method gets its power, you will certainly have dealt with every eventuality related to your problem. Basically what is left is to perform what would have been the final step of your old recursive argument. This requires some familiarity with first-order logic and model theory, so that you know how far you can go with your arguments. But the time spent learning that will pay itself back handsomely in time saved later.

A good place to start learning this is Alan's first introduction, [7], which has an elementary proof of Arhangel'skiĭ's theorem that one should put next to the Pol–Shapirovskiĭ argument to see the difference between the 'standard' and the elementarity mindset. A later survey, [14], gives more applications of the latter.

2. Remote points

If X is completely regular then it is dense in βX, so every point of X^* lies close to X; however some points lie closer to X than others. One can formulate degrees of closeness by stipulating that the point belongs to the closure of a topologically small subset of X. Thus, for example, $p \in X^*$ is *near* if $p \in \mathrm{cl}_\beta D$ for some closed and discrete subset of X; other variants can be obtained by using relatively discrete subsets, scattered subsets, and nowhere dense sets. The negation of the last notion has proved to be very fruitful: call $p \in X^*$ a *remote point of* X if $p \notin \mathrm{cl}_\beta A$ for all nowhere dense subsets of X. Remote points were introduced by Fine and Gillman in 1962 who proved that the Continuum Hypothesis (abbreviated CH) implies that the real line $\mathbb{R}$ has a remote point. Actually, their proof applies to every separable and non-pseudocompact space. Around 1980 van Douwen, and independently Chae and Smith, proved in ZFC that every non-pseudocompact space with countable π-weight has remote points. That there are spaces without remote points, was demonstrated by van Douwen and van Mill. In 1982, Alan took over the research on remote points completely, leaving absolutely nothing for his competitors (see [1–3,5,6,8,20,28,36]). He became the world's expert on remote points. He substantially improved the results of van Douwen, and Chae and Smith by showing that every non-pseudocompact ccc space of π-weight at most ω_1 has a remote point, and that under CH the bound ω_1 is not optimal. The fruits of remote points are manifold. The points themselves were used in 'honest' proofs of non-homogeneity of certain Čech–Stone remainders: for example $\mathbb{Q}^*$ is extremally disconnected at each remote point but not at other points. The techniques developed and used for their construction have found many applications too. Alan's proof that $\omega \times 2^\kappa$ has remote points gave new insight in the structure of the partial order that adds Cohen reals: a remote point, seen as a clopen