Topology and its Applications 214 (2016) 100–108

Contents lists available at ScienceDirect

Topology and its Applications

www.elsevier.com/locate/topol

Induced mappings on symmetric products of continua

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ARTICLE INFO

Article history:
Received 22 March 2016
Received in revised form 23 September 2016
Accepted 25 September 2016
Available online 29 September 2016

MSC:
primary 54B20, 54C10
secondary 54F15

Keywords:
Continuum
Symmetric product
Induced mapping

ABSTRACT

Given a continuum X and a positive integer n, let $F_n(X)$ be the hyperspace of all nonempty subsets of X having at most n points. Given a mapping $f : X \to Y$ between continua, we study the induced mapping $f_n : F_n(X) \to F_n(Y)$ given by $f_n(A) = f(A)$ (the image of A under f). In this paper, we prove some relationships among the mappings f and f_n for the following classes of mappings: almost open, almost monotone, atriodic, feebly monotone, local homeomorphism, locally confluent, locally weakly confluent, strongly monotone and weakly semi-confluent.

1. Introduction

A *continuum* is a nonempty compact connected metric space. Given a continuum X and a positive integer n, the symbol $F_n(X)$ denotes the hyperspace of all nonempty subsets of X having at most n points, topologized with the Hausdorff metric (see [1, p. 1]).

In this paper, a *mapping* means a continuous function. Each mapping between continua $f : X \to Y$ induces a mapping f_n between $F_n(X)$ and $F_n(Y)$ given by $f_n(A) = \{f(a) : a \in A\}$ (see [2, Corollary 1.8.23, p. 65]).

Let $\mathcal{M}$ be a class of mappings between continua. A general problem is to find all possible relationships among the following three statements:

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http://dx.doi.org/10.1016/j.topol.2016.09.011
0166-8641/

(1) $f \in \mathcal{M}$.
(2) $f_n \in \mathcal{M}$ for some $n \geq 2$.
(3) $f_n \in \mathcal{M}$ for every $n \geq 2$.

In this paper we study the interrelations among the statements (1)–(3), for the following classes of mappings: almost open, almost monotone, atriodic, feebly monotone, local homeomorphism, locally confluent, locally weakly confluent, strongly monotone and weakly semi-confluent.

Readers especially interested in this problem are referred to [3–5].

2. Preliminaries

The set of all positive integers will be denoted by $\mathbb{N}$.

For a set Z and $n \in \mathbb{N}$, the cardinality of Z will be denoted by $|Z|$, the collection of all subsets of Z having exactly n elements will be represented by $[Z]^n$ and the symbol $\mathfrak{F}(Z)$ will represent the family of all nonempty finite subsets of Z.

Let X be a continuum and let A be a subset of X. We will use $\mathrm{cl}_X(A)$ and $\mathrm{int}_X(A)$ to represent its closure in X and its interior in X, respectively, and the set of all components of A in X will be represented by $\mathrm{comp}_X(A)$. We will consider the following families of sets:

$$\begin{aligned}\tau_X &= \{A \subseteq X : A \text{ is open subset of } X\},\\ 2^X &= \{A \subseteq X : A \text{ is closed subset of } X \text{ and } A \neq \emptyset\},\\ C(X) &= \{A \in 2^X : A \text{ is connected}\} \text{ and}\\ C_0(X) &= \{A \in C(X) : \mathrm{int}_X(A) \neq \emptyset\}.\end{aligned}$$

Given a family $\mathcal{U}$ of subsets of X and $n \in \mathbb{N}$, we define

$$\langle \mathcal{U} \rangle_n = \left\{A \in F_n(X) : A \subseteq \bigcup \mathcal{U} \text{ and } A \cap U \neq \emptyset \text{ for every } U \in \mathcal{U}\right\}.$$

It is known that the family $\{\langle \mathcal{U} \rangle_n : \mathcal{U} \in \mathfrak{F}(\tau_X)\} - \{\emptyset\}$ is a basis of a topology for $F_n(X)$ (see [1, Theorem 0.11, p. 9]) called the *Vietoris Topology*. It is known that the Vietoris topology and the topology induced by the Hausdorff metric coincide (see [1, Theorem 0.13, p. 10]).

For a subset U of X and $n \in \mathbb{N}$, let $U_n^+ = \langle \{U\} \rangle_n = \{A \in F_n(X) : A \subseteq U\}$ and $U_n^- = \langle \{X, U\} \rangle_n = \{A \in F_n(X) : A \cap U \neq \emptyset\}$. Thus, when U is open (closed, resp.), then U_n^+ and U_n^- are open (closed, resp.) subsets of $F_n(X)$.

It is known that if $\mathcal{U} \in \mathfrak{F}(C(X))$ is such that $\langle \mathcal{U} \rangle_n \neq \emptyset$, then $\langle \mathcal{U} \rangle_n$ is a subcontinuum of $F_n(X)$ (see [6, Lemma 1, p. 230]). In particular, $\{K_n^+ : K \in C(X)\} \subseteq C(F_n(X))$.

Lemma 2.1. *Let X be a continuum and let $n \geq 2$. Then, $\{K_n^+ : K \in C_0(X)\} \subseteq C_0(F_n(X))$.*

Proof. Let $K \in C_0(X)$. Then, $\mathrm{int}_X(K) \neq \emptyset$ and so, $(\mathrm{int}_X(K))_n^+ \in \tau_{F_n(X)} - \{\emptyset\}$. Now, the inclusion $(\mathrm{int}_X(K))_n^+ \subseteq K_n^+$ implies that $\mathrm{int}_{F_n(X)}(K_n^+) \neq \emptyset$. By the previous paragraph, we obtain that $K_n^+ \in C_0(F_n(X))$. □

For the sake of simplicity, if there is no risk of confusion, we will use the symbol $\langle \mathcal{U} \rangle$, U^+ and U^- instead of $\langle \mathcal{U} \rangle_n$, U_n^+ and U_n^-, respectively.

Proposition 2.2. *Let $f : X \to Y$ be a mapping between continua, let $\mathcal{B} \in \mathfrak{F}(2^Y)$, let $n \geq 2$ and let C be a subset of $f_n^{-1}(\langle \mathcal{B} \rangle)$. If $\mathcal{B}$ is a pairwise disjoint collection and $|\mathcal{B}| \leq n$, then $\mathsf{C} \in \mathrm{comp}_{F_n(X)}(f_n^{-1}(\langle \mathcal{B} \rangle))$*