Topology and its Applications 214 (2016) 186–191

Contents lists available at ScienceDirect

Topology and its Applications

www.elsevier.com/locate/topol

2-Adjacency between pretzel links and the trivial link ☆

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ARTICLE INFO

Article history:
Received 1 August 2016
Received in revised form 25 October 2016
Accepted 25 October 2016
Available online 29 October 2016

MSC:
57M25

Keywords:
2-Adjacency
Pretzel link
Conway polynomial
Jones polynomial

ABSTRACT

We study 2-adjacency between a 3-strand pretzel link with two components and the trivial link, and whether the trivial knot is 2-adjacent to a pretzel knot. By investigating mainly the coefficients of their Conway polynomials, Jones polynomials and etc., we show that any nontrivial pretzel link with two-components is not 2-adjacent to the trivial link, and vice versa. Moreover, we also show that the trivial knot is not 2-adjacent to any nontrivial pretzel knot.

1. Introduction

A link (knot) L is called 2-adjacent to a link (knot) W, if L admits a projection D containing two crossings c_1, c_2 such that switching any $0 < s \leq 2$ of them yields a projection of W [1,10,11]. Its properties can be found in [10,11]. The 2-adjacency of classical pretzel knots has been proven that only the trefoil knot and the figure-eight knot are 2-adjacent to the trivial knot [8]. In this paper, we will study the 2-adjacent relation between a two-component pretzel link and an unlink, whether the trivial knot is 2-adjacent to a pretzel knot. We conclude the following theorem.

Theorem 1.1. *Any nontrivial pretzel link with two components is not 2-adjacent to the trivial link and vice versa.*

Theorem 1.2. *The trivial knot is not 2-adjacent to any nontrivial pretzel knot.*

☆ The Project Supported by Zhejiang Provincial Natural Science Foundation of China (Grant No. LY12A01025).
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http://dx.doi.org/10.1016/j.topol.2016.10.010

In the sequel, we assume that the reader is familiar with the definitions and the basic properties of the Conway polynomial $\nabla(*)$, the Jones polynomial $V(*;t)$, and the Homfly polynomial $P(*;l,m)$ of links (knots). They can be found in [3–6].

Convention: We always assume that $L = L_1 \bigcup L_2$ is 2-adjacent to an oriented link $W = W_1 \bigcup W_2$ (here L_2 and W_2 are empty for the case of knots, i.e. $L = L_1$ and $W = W_1$) and the two related crossings are denoted by c_1 and c_2 respectively. Since L is 2-adjacent to W, so there is a 2-adjacent diagram of L denoted by $D(c_1, c_2)$ and $D(oc_1, oc_2)$ is a diagram obtained from $D = D(c_1, c_2)$ by opening c_1 and c_2 respectively. The sign of c_1 (resp. c_2) is denoted by α (resp. β) and $c_1 \in L_1$. Hence, L_2 is W_2. $a_n(G)$ denotes the coefficient of z^n in the Conway polynomial of G. Moreover, $lk(G)$ denotes the total linking number of a link G (see p. 133 in [6]).

2. Preliminary

Definition 2.1. ([5]) A pretzel link is denoted by $P(-\eta b; q_1, \cdots, q_n)$ with $b \geq 0$, $|q_i| > 1$, $(i = 1, \cdots, n)$ and $\eta = \pm 1$. Here $-\eta b$ denotes b strands which each strand has only one crossing with sign η. The condition for $P(-\eta b; q_1, \cdots, q_n)$ to be a knot is that either $n \geq 0$ and all of the q_i's and $n + b$ are odd or just one of the q_i's is even. We call it a pretzel knot of odd type in the former case, and a pretzel knot of even type in the latter case. Fig. 1 gives an example.

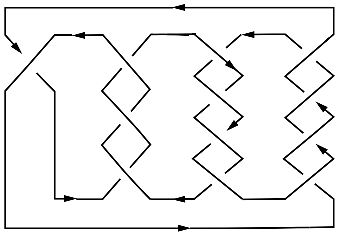

Fig. 1. $P(1, -3, -4, 4) = P(-1; -3, -4, 4)$.

It is clear that a link (knot) is 2-adjacent to an unlink (unknot) if and only if its mirror image is and if the unlinking (unknotting) number of a pretzel link (knot) is one, then it has only three strands [7] and two of them have an even number of crossings for the pretzel link.

Proposition 2.2. ([8–10]) *If the notations and the conditions are as the convention, then $lk(L) = lk(W)$ and*

$$\nabla(L) = \alpha\beta z^2 \nabla(D(oc_1, oc_2)) + \nabla(W). \tag{2.1}$$

(1) *If $a_3(L) \neq a_3(W)$, then $D(oc_1, oc_2)$ is a two-component link.*
(2) *If $a_3(L) = a_3(W)$, then either $lk(L) = 0$ and $D(oc_1, oc_2)$ is a link with two components or $D(oc_1, oc_2)$ is a link with four components.*
(3) *$D(oc_1, oc_2)$ is a link with four components if and only if $a_2(L_1) = a_2(W_1)$; $D(oc_1, oc_2)$ is a link with two components if and only if $a_2(L_1) = \alpha\beta + a_2(W_1)$. Moreover, for the second case,*

$$\nabla(L_1) = \alpha\beta z^2 \nabla(\widehat{D}(oc_1, oc_2)) + \nabla(W_1), \tag{2.2}$$

where $\widehat{D}(oc_1, oc_2)$ is obtained by opening c_1, c_2 from L_1.
(4) *If L is the trivial link and is 2-adjacent to W, then $D(oc_1, oc_2)$ has four components. Furthermore, if $a_4(W_1) = 0$, then $lk(\widehat{D}(oc_1, oc_2)) = 0$.*