Topology and its Applications 212 (2016) 19–28

Contents lists available at ScienceDirect

Topology and its Applications

www.elsevier.com/locate/topol

Tight planar contact manifolds with vanishing Heegaard Floer contact invariants

James Conway [a,*], Amey Kaloti [b], Dheeraj Kulkarni [c]

[a] Department of Mathematics, University of California, Berkeley, United States
[b] Department of Mathematics, University of Iowa, United States
[c] Department of Mathematics, RKM Vivekananda University, India

ARTICLE INFO

Article history:
Received 8 June 2016
Accepted 31 August 2016
Available online 5 September 2016

Keywords:
Contact geometry
Open book decomposition

ABSTRACT

In this note, we exhibit infinite families of tight non-fillable contact manifolds supported by planar open books with vanishing Heegaard Floer contact invariants. Moreover, we also exhibit an infinite such family where the supported manifold is hyperbolic.

1. Introduction

Many techniques have been developed to determine whether a given contact 3-manifold is tight. As each one was developed, the question arose as to whether a given property is equivalent to tightness. Fillability was the first widely-used tool to prove tightness, but tight contact manifolds were found by Etnyre and Honda that were not fillable [8]. Ozsváth and Szabó then developed Heegaard Floer theory, which very promisingly could prove tightness in many cases where the manifold was not fillable. However, many tight contact manifolds with vanishing Heegaard Floer contact invariant have been discovered, see [11,12].

On the side of positive results, Honda, Kazez, and Matić [16] proved that for contact manifolds supported by open books with pages of genus one and connected binding, tightness is equivalent to the non-vanishing of the Heegaard Floer contact invariant. For contact manifolds supported by open books with planar pages, the first tight but non-fillable examples came from applying [21, Corollary 1.7] to examples in [13,22]. They proved tightness by showing that the Heegaard Floer contact invariant does not vanish. The following question, however, remained.

* Corresponding author.
E-mail addresses: conway@berkeley.edu (J. Conway), amey-kaloti@uiowa.edu (A. Kaloti), dheeraj.kulkarni@gmail.com (D. Kulkarni).

http://dx.doi.org/10.1016/j.topol.2016.08.029
0166-8641/

Question 1.1. For contact manifolds supported by open books with planar pages, is tightness equivalent to the non-vanishing of the Heegaard Floer contact invariant?

We provide a negative answer to this question. In particular, we construct infinite families of tight contact manifolds supported by planar open books where capping off a binding component recovers a given overtwisted manifold. Since capping off a binding component of an open book is equivalent to admissible transverse surgery on a binding component, we have the following.

Theorem 1.2. *Given any overtwisted contact manifold (M, ξ), there exist infinite families of tight, planar, non-fillable contact manifolds with vanishing Heegaard Floer contact invariant, on which admissible transverse surgery on a link in these manifolds recovers (M, ξ).*

We hope that these examples will be useful in further exploring characterizations of tightness, and in particular will provide computable examples of boundary cases. Indeed, these examples have already been used to investigate possible new invariants of contact structures coming from Heegaard Floer Homology being developed by Kutluhan, Matić, Van Horn-Morris, and Wand [20] and Baldwin and Vela-Vick [3]. These invariants are interesting when the Heegaard Floer contact invariant vanishes, as in the examples from Theorem 1.2.

The only infinite family of tight, non-fillable contact manifolds that are hyperbolic has been produced by Baldwin and Etnyre [2]. Their examples are supported by open books with pages of genus one. In addition, their construction requires throwing out a finite number of unspecified members of their infinite family that may not be hyperbolic. We produce an infinite family of such manifolds with planar supporting open books, all of whom are hyperbolic. To this end, let S denote the surface shown in Fig. 1. Let $\mathbf{v} = (p, n_1, n_2, n_3, n_4)$ be a 5-tuple of integers, and let $\phi_{\mathbf{v}} = \tau_\alpha^{-n_1-1}\tau_\beta^p\tau_{B_1}^{n_1}\tau_{B_2}^{n_2}\tau_{B_3}^{n_3}\tau_{B_4}^{n_4}$ be a diffeomorphism of S. We show the following.

Theorem 1.3. *Let $(M_{\mathbf{v}}, \xi_{\mathbf{v}})$ be the contact manifold supported by the open book $(S, \phi_{\mathbf{v}})$. Then $(M_{\mathbf{v}}, \xi_{\mathbf{v}})$ is universally tight, not fillable, has vanishing Heegaard Floer contact invariant, and is hyperbolic, for $p \geq 1$, and $n_i \geq 6$ for each i.*

This paper is organized as follows. In Section 2 we recall the definitions and properties of open books, fillability, the Heegaard Floer contact invariant, and transverse surgery. We prove our results in Section 3.

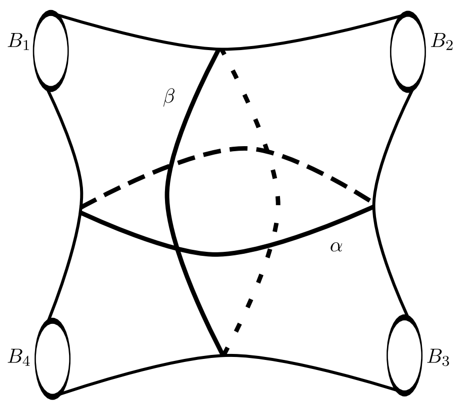

Fig. 1. Surface S used in constructing $(S, \phi_{\mathbf{v}})$, with α and β curves indicated.