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Strong Γ-ideal convergence in a probabilistic normed space

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ABSTRACT

In this paper, we investigate some properties of the set of all strong ideal cluster points of a sequence in a probabilistic normed (PN) space. We also introduce the concept of strong Γ-ideal convergence in a PN space, and investigate some of its main properties.

1. Introduction

Probabilistic normed (PN) spaces are special cases of probabilistic metric (PM) spaces [16]. A PN space is a generalization of an ordinary normed vector space and, in a PN space, the norms of the vectors are represented by probability distribution functions. For instance, if p is an element of a PN space, then its norm is usually denoted ν_p, and the value $\nu_p(x)$ is interpreted as the probability that the norm of p is smaller than x, where x is an extended real number.

PN spaces were first introduced by Šerstnev in [21]. In 1993, Alsina et al. [1] presented a new definition of a PN space which includes the definition of Šerstnev [21] as a special case. In this study, we will adopt this new definition. Also, following [1], many papers (for instance, [2,9–12]) investigating the properties of

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PN spaces have appeared. A detailed history and the development of this subject up to 2006 can be found in [17].

One of the basic concepts in the theory of PN spaces is the concept of distributional boundedness (D-boundedness, briefly). This notion has a probabilistic nature and it was investigated by Lafuerza Guillén et al. [12] in the setting of PN spaces via the concept of probabilistic radius.

On the other hand, relying on the concept of ideal convergence (a generalization of statistical convergence) defined in [7], Şençimen and Pehlivan [18] introduced the concept of strong ideal convergence in a PM space endowed with the strong topology. Later, in [19], they introduced the concept of a statistically D-bounded sequence in a PN space, depending on the concepts of real statistically bounded sequence introduced in [6,23] and D-bounded set introduced in [12]. Statistical convergence [4,22] and statistical boundedness are more general tools than their ordinary counterparts. The statistical convergence was also investigated in a general Hausdorff topological space [13].

In this paper, first, we compare some of the results related to the concept of statistical cluster point introduced by Fridy [5] and developed by Pehlivan et al. [15], with our results obtained in the more general probabilistic setting. Next, using the concept of Γ-statistical convergence introduced in [15] for sequences in finite dimensional spaces, we introduce the concept of strong Γ-ideal convergence in a PN space endowed with the strong topology. All the new definitions and results presented in our paper will be based on, and a generalization of the ones given in [15] and [20].

Furthermore, since the concept of statistical cluster point of a sequence is very important in the study of optimal paths and turnpike theory (see [14] and [24]), the results presented in our study could be useful theoretical tools for the study of optimal paths in a PN space.

2. Preliminaries

First, we recall some of the basic concepts related to the theory of PN spaces. For more details, we refer to [12] and [16].

A *distribution function* is a nondecreasing function F defined on $[-\infty, +\infty]$, with $F(-\infty) = 0$ and $F(\infty) = 1$. The set of all distribution functions that are left-continuous on $(-\infty, \infty)$ is denoted by Δ. Moreover, for any a in $[-\infty, +\infty]$, the unit step functions in Δ are defined by

$$\varepsilon_a(x) = \begin{cases} 0, & -\infty \le x \le a \\ 1, & a < x \le \infty \end{cases} \quad \text{for } -\infty \le a < \infty,$$

$$\varepsilon_\infty(x) = \begin{cases} 0, & -\infty \le x < \infty \\ 1, & x = \infty. \end{cases}$$

The distance $d_L(F, G)$ between two elements F and G in Δ is defined as the infimum of all numbers $h \in (0, 1]$ such that the inequalities

$$F(x - h) - h \le G(x) \le F(x + h) + h$$

and

$$G(x - h) - h \le F(x) \le G(x + h) + h$$

hold for every $x \in \left(-\frac{1}{h}, \frac{1}{h}\right)$. d_L is called the *modified Lévy metric.*

A *distance distribution function* is a nondecreasing function F defined on $[0, +\infty]$ that satisfies $F(0) = 0$ and $F(\infty) = 1$, and is left-continuous on $(0, \infty)$. The set of all distance distribution functions is denoted by Δ^+.