Topology and its Applications 201 (2016) 217–234

Contents lists available at ScienceDirect

Topology and its Applications

www.elsevier.com/locate/topol

Inverse systems and inverse limits in the category of plain textures

Filiz Yıldız [1]

Hacettepe University, Faculty of Science, Mathematics Department, Beytepe, Ankara, Turkey

ARTICLE INFO

Article history:
Received 19 November 2014
Accepted 7 May 2015
Available online 23 December 2015

MSC:
primary 46A13, 18A30, 46M40, 54B30
secondary 03E04, 06D10, 03E20

Keywords:
Inverse system
Inverse (projective) limit
Directed set
Cofinal
Direct limit
Thread
Bonding map
Limiting projection
Texture
Plain texture
Real texture
Category
Contravariant functor
Equalizer
Isomorphic categories

ABSTRACT

This paper considers the notions of inverse system and inverse limit in a category consisting of special type textures. By restricting ourselves to especially working within plain textures as a major type of textures, we constructed many elementary properties and the various aspects of inverse systems, and their limits. Additionally, we proved a representation theorem which states infinite product of plain textures can be represented as the inverse limit of an inverse system consisting of finite products of that textures.

1. Introduction and preliminaries

One of the principle aims of present work is to construct the foundations of a corresponding theory of inverse systems and their inverse limits, in the category of plain textures which are special types of textures. The other is to express any infinite textural product of the objects in that category as the inverse limit of an inverse system consisting of finite textural products of those objects.

E-mail address: yfiliz@hacettepe.edu.tr.

[1] The author acknowledges support under Research Project Number 013 D11 601 001, awarded by the Hacettepe University Scientific Research Unit.

http://dx.doi.org/10.1016/j.topol.2015.12.038
0166-8641/

Accordingly, this paper consists of five sections and the layout of the paper is as follows:

After presenting some preliminary material required for the paper, in Section 2 mainly, we introduce the category **ifPTex** of plain textures, that is the textures closed under arbitrary unions, and the point functions satisfying a compatible condition, called *ω-preserving*. Following this, Section 3 begins by describing the notion of *inverse system* peculiar to the category **ifPTex** and also contains another several definitions, examples and results that are important in their own right, and which will also be needed later on. Specifically, this section ends with some considerations about the mappings between inverse systems.

Besides, in Section 4 after introducing the notion of *inverse limit* for any inverse system in **ifPTex** and the other concepts related with the inverse limits inside **ifPTex**, many useful examples to illustrate the nature of inverse limits are presented. Followed by required theorems and results for the rest of paper are mentioned in a categorical setting insofar as plain textures are concerned. Finally, Section 4 ends by representing any infinite (textural) product of objects in the category **ifPTex** as the limit of an inverse system consisting of finite products of these objects.

Following that, Section 5 as the last part of the paper gives a conclusion about the presented work.

It should be noted that here no attempt will be made at the inverse systems of general textures. In this paper our approach is designed to permit a direct transition from the inverse systems of plain textures to the inverse systems of general textures which need not to be plain and to the inverse systems of ditopological texture spaces (see [12–15] and [17–20]) although we will not consider them in the present context.

Motivation and background material specific to the main topic of this paper may be found in [4–8] and [11–20]. Especially, a significant reference in the general field of inverse system theory is [7].

Incidentally, the reader may consult [10] for terms from lattice theory not mentioned here. In addition our standard reference for notions and results from category theory is [1] and if **A** is a category, Ob **A** will denote the class of objects and Mor **A** the class of morphisms of **A**.

The remainder of this introduction will be given over to some background material. Hence, we will conclude this section by recalling some preliminary information and results that will enable a casual reader to follow the general ideas presented here.

Inverse systems and (inverse) limits The origins of the study of inverse limits date back to the 1920's and 1930 's. Classical theory of inverse systems and inverse limits are important in the extension of homology and cohomology theory from simplicial objects to the more general Čech theory, applicable to a wide class of spaces. Let us note that the notions of inverse system and (inverse) limit of an inverse system can be defined without a topology on the sets; so these notions are also studied in algebra and analysis. An exhaustive discussion of inverse systems which are in some classical categories such as **Set**, **Top** and **Rng** was presented by [7].

The categorical dual of an inverse limit is a *direct limit (or inductive limit)* (see [6,7]). More general concepts are *limits* and *colimits* in category theory [1] and many basic constructions in mathematics can be described as limits or dually as colimits. Incidentally, inverse limits are limits while direct limits are colimits.

Returning to work at the moment, we will give a detailed analysis of the theory of inverse systems and inverse limits insofar as the category of plain textures is concerned, particularly. Also, no attempt will be made at the notions of direct systems of textures even plain ones, and their (direct) limits.

Now let us briefly present some information about *texture theory*:

Textures The notion of *texture* (also called fuzzy structure) first arose in connection with the representation of Hutton algebras and lattices of $\mathbb{L}$-fuzzy sets in a point-based setting but it is becoming increasingly clear that in a more general vein they provide a fruitful setting for the development of concepts and techniques which may be used where (set) complementation is not available. The reason is that textures are generally not closed under set-complementation.