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Polynomial invariants of a link with local symmetry

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ABSTRACT

Let D be a link diagram and T a 4-tangle. By replacing each crossing of D by T, we get a new diagram $D \otimes T$, called a link diagram with local symmetry or tensor product of D and T. In this paper, we will study polynomial invariants of the link diagram $D \otimes T$ with local symmetry in terms of D and T, and as an application, we will study the adequacy of $D \otimes T$.

1. Introduction

Let D be a diagram of a link L. A *state* s of D is an assignment of characters A and B to each crossing of D as Fig. 1. Given a state s of D, we can splice each vertex by giving either A-*split* or B-*split*, see Fig. 1. Clearly, there are $2^{c(D)}$ different states where $c(D)$ is the number of crossings in D.

For a given state s of D, let $a(s)$ and $b(s)$ denote the numbers of the character A and the character B which are assigned to crossings of D respectively, and let $|s|$ denote the number of loops of the resulting diagram obtained from D by splicing all crossings of D according to its state. The *Kauffman bracket polynomial* $< D > \in \mathbb{Z}[A, B, d]$ of the link diagram D is given by

$$< D >= \sum_s A^{a(s)} B^{b(s)} d^{|s|-1}.$$

It is well-known that the Kauffman bracket polynomial $< D >$ is a regular isotopy invariant if $B = A^{-1}$ and $d = -A^2 - A^{-2}$, and that

$$V_L(A) = (-A)^{-w(D)} < D >$$

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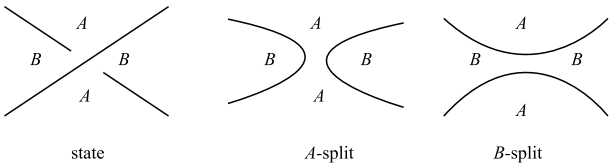

Fig. 1. A-split and B-split.

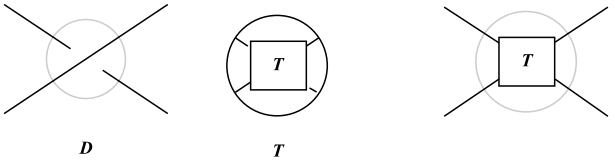

Fig. 2. $D \otimes T$.

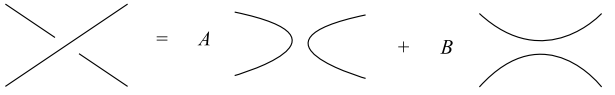

Fig. 3. The skein relation.

is the *Jones polynomial* of L, which is an isotopy invariant of L. Here $w(D)$ denotes the *writhe* of D.

Let D be a link diagram and T a 4-tangle. By replacing each crossing of D by T as in Fig. 2, we get a new diagram $D \otimes T$, called a *link diagram with local symmetry* or *tensor product of D and T*.

Example 1. Let D be the following diagram of the trefoil knot and let T be the diagram of the rational tangle $C(2, -3, 2)$. Then tensor product $D \otimes T$ of D and T is the right diagram of the following figure.

In this paper, we will study the Kauffman bracket polynomial $< D \otimes T >$ and the adequacy of the link diagram $D \otimes T$ with local symmetry.

2. Kauffman bracket polynomial of a link diagram with local symmetry

The bracket polynomial can be characterized by the skein module. The *skein module* $E(A, B, d)$ is the complex vector space generated by all link diagrams with the following relations:

(i) ambient isotopy in the plane;
(ii) $D \cup \bigcirc = d \cdot D$, where D is an arbitrary link diagram and $\bigcirc$ is a simple closed curve bounding a disk in the complement of D;
(iii) The skein relation for Kauffman bracket polynomial (see Fig. 3).