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Moduli spaces of 6 and 7-dimensional complete intersections ☆

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This paper proves the existence of homeomorphic (diffeomorphic) complex 6-dimensional (7-dimensional) complete intersections that belong to components of the moduli space of different dimensions. These results are given as a supplement to earlier result on 5-dimensional complete intersections.

1. Introduction

Narasimhan and Simha [9] proved that the isomorphism classes of complex analytic structures with ample canonical bundle on a topological space form a Hausdorff space. Let $X_n(\underline{d}) \subset \mathbb{C}P^{n+r}$ be a smooth complete intersection of multidegree $\underline{d} = (d_1, \dots, d_r)$, the product $d_1 d_2 \cdots d_r$ is called the total degree, denoted by d. Let $m(X_n(\underline{d}))$ denote the dimension of the moduli space component containing the isomorphism class of $X_n(\underline{d})$, the explicit dimension formula about $m(X_n(\underline{d}))$ is given in [1, Lemma 3].

Libgober and Wood [8] showed the existence of homeomorphic complete intersections of dimension 2 and diffeomorphic ones of dimension 3 which belong to components of the moduli space having different

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dimensions. Brückmann [1,2] showed that there are families of arbitrary length k of complete intersections in $\mathbb{C}P^{4k-2}$ and $\mathbb{C}P^{6k-2}$ (resp. $\mathbb{C}P^{5k-2}$) consisting of homeomorphic complete intersections of dimensions 2 and 4 (resp. diffeomorphic ones of dimension 3) but that belong to components of the moduli space of different dimensions. Keep going along this line, the author [11] proved the existence of any k diffeomorphic complex 5-dimensional complete intersections in $\mathbb{C}P^{7k-2}$ belonging in the moduli space to components with different dimensions. The diffeomorphic complex 5-dimensional complete intersections are based on the following two theorems due to Fang, the author [5, Theorem 1.1] and Traving [6, Theorem A] or [10].

Theorem 1.1 *(Fang–Wang). Two complete intersections $X_n(\underline{d})$ and $X_n(\underline{d}')$ are homeomorphic if and only if they have the same total degree, Pontrjagin classes and Euler characteristics, provided $n = 5, 6, 7$.*

Theorem 1.2 *(Traving). To the prime factorization of total degree $d = \prod_{p \text{ primes}} p^{\nu_p(d)}$, if $\nu_p(d) \geqslant \frac{2n+1}{2(p-1)} + 1$ for all primes p with $p(p-1) \leqslant n+1$ and $n > 2$, then two complete intersections $X_n(\underline{d})$ and $X_n(\underline{d}')$ are diffeomorphic if and only if they have the same total degree, Pontrjagin classes and Euler characteristics.*

The goal of this paper is to give the following main theorem, which is a supplement to the results of Brückmann [1,2] and the author [11].

Theorem 1.3. *For each integer $k > 1$, there exist k homeomorphic (diffeomorphic) complex 6-dimensional (7-dimensional) complete intersections in $\mathbb{C}P^{8k-2}$ ($\mathbb{C}P^{15k-8}$) which lie in different dimensional components of the moduli space.*

These homeomorphic (6-dimensional) or diffeomorphic (7-dimensional) examples are easy to check but hard to happen upon. Computer search, which is more efficient in lower dimensions (e.g. $n \leqslant 5$, cf. [11]), is a good way to obtain these examples. Fortunately, searching of these examples is related to the problem of *Equal products and equal sums of like powers* (cf. [3,4]), which is finding two distinct sets of integers $\{x_1, \ldots, x_r\}$ and $\{y_1, \ldots, y_r\}$, such that $\prod_{i=1}^r x_i = \prod_{i=1}^r y_i$ and $\sum_{i=1}^r x^k = \sum_{i=1}^r y^k$, where $k = k_1, \ldots, k_n$ are specified positive integers. The particular case for the problem of *Equal sums of like powers* when $k = 1, 2, \ldots, n$ is the well-known Tarry–Escott problem. Evidently, according to topological characteristic classes formulas in Section 2, solutions of *Equal products and equal sums of like powers* will offer us multidegrees to make homeomorphic or diffeomorphic complete intersections.

2. Characteristic classes of 6 and 7-dimensional complete intersections

Let $X_n(\underline{d}) \subset \mathbb{C}P^{n+r}$, where $n \geqslant 2$, $\underline{d} = (d_1, \ldots, d_r)$, $d_i \geqslant 2$. As usual, define the power sums

$$s_i = \sum_{j=1}^r d_j^i \text{ for } 1 \leqslant i \leqslant n.$$

Let $x \in H^2(X_n(\underline{d}); \mathbb{Z}) \cong \mathbb{Z}$ be the generator, satisfying

$$x^n \cap [X_n(\underline{d})] = d = d_1 \cdots d_r.$$

Then the Pontrjagin classes and Euler characteristic of $X_n(\underline{d})$ depend only on the dimension n, total degree d and power sums s_i (Please see [7, §7] or [11, §2] for more details).