Topology and its Applications 197 (2016) 102–111

Contents lists available at ScienceDirect

Topology and its Applications

www.elsevier.com/locate/topol

QC-continuity of posets and the Hoare powerdomain of QFS-domains ☆

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ARTICLE INFO

Article history:
Received 10 May 2015
Received in revised form 11 October 2015
Accepted 14 October 2015
Available online 11 November 2015

Keywords:
Quasicontinuous domain
QFS-domain
GCD lattice
QC-continuous poset
Scott topology
Hoare powerdomain

ABSTRACT

In terms of Scott-closed sets, the concept of QC-continuous posets, a generalization of C-continuous lattices is introduced. With this new concept, the following results are obtained: (1) a complete lattice is generalized completely distributive (GCD) iff it is quasicontinuous and QC-continuous; (2) a dcpo L is quasicontinuous (resp., quasialgebraic) iff the Hoare powerdomain $H(L)$ is quasicontinuous (resp., quasialgebraic); (3) the Hoare powerdomain $H(L)$ of a QFS-domain L is still a QFS-domain.

1. Introduction

Domain theory is one of the important research fields of theoretical computer science. Mutual transformations and infiltration of order, topology and logic are the basic features of this theory. To investigate domains, as well as general posets, some intrinsic topologies play important roles. A basic and remarkable result given by Lawson in [11] shows that a dcpo is continuous iff the lattice of its Scott-closed (Scott-open) subsets is a completely distributive lattice (in short, CD lattice). Recently, Ho and Zhao [8] introduced the concept of C-continuous lattices, and showed that for a poset L, the lattice $\sigma^*(L)$ of all Scott-closed subsets of L is always C-continuous and thus proved that L is continuous iff $\sigma^*(L)$ is continuous.

Powerdomain constructions proposed first by Plotkin [15] in domain theory provide mathematical models for semantics of non-deterministic programming languages. One fundamental problem in denotational

☆ Supported by the NSF of China (61472343, 11401514, 61300153, 61271002) and University Science Research Project of Jiangsu Province (15KJD110006).

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http://dx.doi.org/10.1016/j.topol.2015.10.007

semantics is whether there exists a powerdomain construction for a given category of nice enough domains. With this problem, much work on various categories of domains has been done [1,2,4,6,7,20].

To investigate maximal Cartesian closed subcategories of **DOM** of domains and Scott continuous functions, Jung [10] introduced the concept of FS-domains. It was showed in [1] that the Hoare (lower) powerdomain and the Smyth (upper) powerdomain of an FS-domain are also FS-domains. It is well known that RB-domains (retracts of bifinite domains) are FS-domains, but whether the converse is true or not is still unknown. Gierz, Lawson and Stralka [3] introduced quasicontinuous domains, the most successful generalizations of domains. Li and Xu in [13] generalized FS-domains to QFS-domains and proved that QFS-domains share many nice properties of FS-domains. Coincidentally, Goubault-Larrecq in [4] generalized RB-domains to QRB-domains and proved that the countably-based QRB-domains are closed under the probabilistic powerdomain construction. Recently, Lawson and Xi in [12] proved via the Smyth powerdomain that QRB-domains, QFS-domains and Lawson compact quasicontinuous domains are the same. Independently, and slightly earlier, Goubault-Larrecq and Jung [5] derived the same result by showing that QRB-domains, QFS-domains and Lawson compact quasicontinuous domains are the same thing as sober, compact, locally finitary compact spaces. With this result, they proved that QRB-domains are closed under the probabilistic powerdomain construction. These results are rather surprising and give more linkages with order structures and topological properties.

The concept of the Hoare (lower) powerdomain was originally defined for domains, and the Hoare powerdomain of a domain is isomorphic to the family of nonempty Scott-closed subsets of the domain ordered by set-inclusion [2]. Later, the family of nonempty Scott-closed subsets of a dcpo L was simply called the Hoare powerdomain of L, and denoted by $H(L)$. It was known from the work of Yang and Luo [19] that the Hoare powerdomain of a quasicontinuous domain with the least element is a GCD lattice in the sense of Venugopalan [17]. Now questions naturally arise: are the Hoare powerdomains of quasicontinuous domains/QFS-domains still quasicontinuous domains/QFS-domains? In other words, are the classes of quasicontinuous domains/QFS-domains closed under the Hoare powerdomain construction or not?

To answer the above questions, in this paper, we will generalize Ho and Zhao's work in [8] by introducing QC-continuity of a poset and give a new characterization of GCD lattices. As applications of the newly defined QC-continuity, we pithily give affirmative answers to the above questions with newly obtained results and lifting technique.

It should be stated that to answer the above questions, there are also at least two other approaches. One is a direct approach suggested by an anonymous editor for this paper. A sketch of the approach will be given as a remark (Remark 4.10) in Section 4. And another approach given by an anonymous referee is a broad one. The referee generalizes the problems in the frame of locally finitary compact spaces initiated by Isbell [9].

The concept of QC-continuity is unnecessary in answering the above questions, however, it has independent interest and does provide an approach to answer the questions.

2. Preliminaries

Recall some basic concepts and results which will be used in the sequel. Most of them come from [2] and [13].

Let L be a poset. Then L with the dual order is also a poset and denoted by L^{op}. For a poset L and $D \subseteq L$, we use $\vee D$ or $\sup D$ (resp., $\wedge D$ or $\inf D$) to denote the supremum (resp., infimum) of D if they exist. A poset is called a directed complete poset (dcpo, in short) if every directed subset in it has a supremum. A subset $A \subseteq L$ is *Scott-closed* iff (i) $A = {\downarrow}A$, and (ii) for any directed subset $D \subseteq A$, one has $\vee D \in A$ whenever $\vee D$ exists. The collection of all Scott-closed sets of L is denoted by $\sigma^*(L)$. The complements of Scott-closed sets of L form a topology, called the *Scott topology* of L and denoted by $\sigma(L)$. The topology on L generated by $\{L \backslash {\uparrow}x : x \in L\}$ as a subbase is called the *lower topology*, which is denoted by $\omega(L)$. And