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A remark on 2-adjacency relation between Montesinos knots and 2-bridge knots

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ABSTRACT

We show that for every 2-bridge knot K, there is an infinite family of Montesinos knots $\{K_q\}$ such that each K_q is 2-adjacent to K. We also give a new example of 2-adjacency relation between 2-bridge knots.

1. Introduction

Let K and K' be knots in S^3. K is called *n-adjacent* to K' for some $n \in \mathbb{N}$, if K admits a diagram containing n (generalized) crossings such that changing any non-empty subset of them yields a diagram of K' [1,5]. In [9,11], and [12], the author studied n-adjacency relation between 2-bridge knots for $n = 2, 3$ using Dehn surgery techniques in detail. Note that 2-adjacency relation between Montesinos knots is still not determined in general.

Recall that a Montesinos knot or link $M((\alpha_1, \beta_1), \dots, (\alpha_t, \beta_t))$ is defined to be a knot or link connecting t rational tangles of slope from $r_1 = \dfrac{\beta_1}{\alpha_1}$ through $r_t = \dfrac{\beta_t}{\alpha_t}$ as indicated in Fig. 1 (α_i and β_i are coprime integers). It is well-known that if $t \le 2$, it is a 2-bridge knot or link. For a reference, see [2] and [3].

In [10], the author studied 2-adjacency relation between Montesinos knots and the trivial knot. In the current short note, we show that for every 2-bridge knot K, there is an infinite family of Montesinos knots $\{K_q\}$ such that each K_q is 2-adjacent to K, and we also present a new example of 2-adjacency relation between 2-bridge knots.

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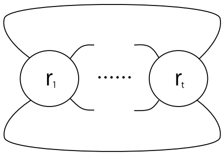

Fig. 1. Montesinos knots.

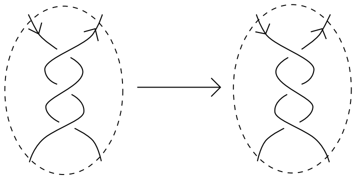

Fig. 2. A generalized crossing change of order -3.

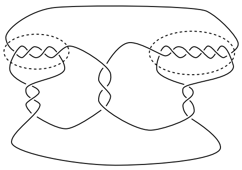

Fig. 3. $MK_{(q,d_1,d_2,\varepsilon_1,\varepsilon_2)} = M((2d_1q+\varepsilon_1, 2d_1), (q, -1), (2d_2q+\varepsilon_2, 2d_2))$, where $q = 3, d_1 = 2, d_2 = -3$ and $\varepsilon_1 = \varepsilon_2 = +1$.

2. Main theorem

Let K be an oriented knot, link, or tangle in S^3. A crossing disk for K is an embedded disk $D \subset S^3$ such that K intersects $\operatorname{int}(D)$ twice with zero algebraic intersection number. The boundary of a crossing disk is called a crossing circle. Then, *a generalized crossing change of order* $d \in \mathbb{Z}$ on K is achieved by $-\frac{1}{d}$-Dehn surgery on a crossing circle and results in introducing d full twists to K at the crossing disk (see Fig. 2). A *generalized crossing* of order d on a diagram of K is a set C of d twist crossings on two strings that inherit opposite orientations of K. If K' is obtained from K by switching all the crossings in C simultaneously, then K' is the result of a generalized crossing change of order d on K. Note that if $|d| = 1$, K and K' differ by an ordinary crossing change while if $d = 0$, then $K = K'$. Throughout this paper, we assume that $d \neq 0$.

Recall that K is 2-adjacent to K', if there are two generalized crossings of order d_1 and d_2 in a diagram of K such that correspondingly, both each generalized crossing change and simultaneous generalized crossing changes yield a diagram of K'.

Example 1. ([10]) For an integer q, non-zero integers d_1, d_2 and $\varepsilon_1 = \pm 1$, $\varepsilon_2 = \pm 1$, let $MK_{(q,d_1,d_2,\varepsilon_1,\varepsilon_2)}$ be the Montesinos knot $M((2d_1q+\varepsilon_1, 2d_1), (q, -1), (2d_2q+\varepsilon_2, 2d_2))$ (see Fig. 3). It is easily observed that the generalized crossings of order $\varepsilon_1 d_1$ and $\varepsilon_2 d_2$ in the dotted circles for $MK_{(q,d_1,d_2,\varepsilon_1,\varepsilon_2)}$ as shown in Fig. 3 make $MK_{(q,d_1,d_2,\varepsilon_1,\varepsilon_2)}$ 2-adjacent to the trivial knot.

Lemma 2. *Suppose* $r = \dfrac{2dn^2}{2dmn+\varepsilon}$ *for some coprime integers* m, n, *a non-zero integer* d *and* $\varepsilon = \pm 1$. *Further, for* $\dfrac{n}{m} = [a_0, a_1, \ldots, a_k] = a_0 + \cfrac{1}{a_1 + \cdots + \cfrac{1}{a_k}}$, *if* k *is odd (resp. even), let* $\epsilon = 1$ *(resp.* -1*). Then, a rational tangle of slope* $\dfrac{0}{1}$ *is obtained from a rational tangle of slope* r *by a generalized crossing change of order* $\epsilon\varepsilon d$.