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Syndetic sensitivity in semiflows

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ABSTRACT

The original Devaney's definition of chaos from 1989 has three conditions, one of which, namely sensitivity, turned out to be redundant. This was first proved by Banks, Brooks, Cairns, Davis and Stacey in 1992. A stronger version of that was proved by Subrahmonian Moothathu in 2007. Another version, independent from Moothathu's, was proved by Wang, Long and Fu in 2012. These statements are corollaries of the main theorem of our paper, which is formulated in the context of an arbitrary acting abelian topological semigroup.
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1. Notation and preliminaries

We always assume that T is an arbitrary abelian unital topological semigroup (i.e., an abelian topological monoid), X is a metric space and (T, X, π), also denoted (T, X), is a unital semiflow (shortly semiflow). For $t \in T$ the transition map $x \mapsto \pi(t, x)$ is denoted by π_t. The orbit of a point $x \in X$ is denoted by Tx or by $\mathcal{O}(x)$. The set of elements of T that fix an element $x \in X$ is denoted by $\mathrm{Fix}(x)$. It is a closed unital subsemigroup of T.

For any unexplained notion the reader can consult [4].

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2. Syndetic sets, periodicity and transitivity

Definition 2.1. A subset A of T is said to be *syndetic* if there is a compact subset K of T such that for every $t \in T$ the translate $t + K$ intersects A. The compact set K is called a *compact corresponding to* A.

Definition 2.2. A point x in a semiflow (T, X) is called *periodic* if there is a syndetic unital subsemigroup S of T such that $S.x = \{x\}$. Equivalently, $\mathrm{Fix}(x)$ is a syndetic subset of T.

This definition is common but not entirely universal. In some papers periodic points are called "minimal points." For discrete semigroups, "periodic orbit" is often taken to simply mean finite orbit. A different, but not very natural, definition of periodicity was considered in [11].

Proposition 2.3. *Let $x \in X$ be a periodic point in a semiflow (T, X). Then:*

(a) *For every $x' \in \mathcal{O}(x)$, $\mathcal{O}(x') = \mathcal{O}(x)$.*
(b) *For every periodic $x' \in X$, either $\mathcal{O}(x) = \mathcal{O}(x')$ or $\mathcal{O}(x) \cap \mathcal{O}(x') = \emptyset$.*
(c) *For every $x' \in \mathcal{O}(x)$, $\mathrm{Fix}(x') = \mathrm{Fix}(x)$. In particular, all points from $\mathcal{O}(x)$ are periodic.*
(d) *If K is a compact subset of T corresponding to $S = \mathrm{Fix}(x)$, then $\mathcal{O}(x) = K.x$. In particular, $\mathcal{O}(x)$ is a compact subset of X.*
(e) *Suppose X is a metric space and $X = \mathcal{O}(x)$. Then (T, X) is a compact, minimal, uniformly equicontinuous semiflow.*

Proof. Let $S = \mathrm{Fix}(x)$ and let K be a compact subset of T whose every translate $t+K$, $t \in T$, intersects S.

(a) Let $x' = t'.x$ for some $t' \in T$. Then there is a $k' \in K$ such that $t' + k' \in S$. Hence $k'.(t'.x) = x$, i.e., $k'.x' = x$. Hence $x \in \mathcal{O}(x')$ and so $\mathcal{O}(x) \subset \mathcal{O}(x')$. Since also $x' \in \mathcal{O}(x)$, we have $\mathcal{O}(x') \subset \mathcal{O}(x)$. Thus $\mathcal{O}(x') = \mathcal{O}(x)$.
(b) Follows from (a).
(c) Let $x' = t'.x$ for some $t' \in T$. Then for any $s \in S$, $s.x' = t'.(s.x) = t'.x = x'$, hence $\mathrm{Fix}(x) \subset \mathrm{Fix}(x')$. By symmetry, $\mathrm{Fix}(x') \subset \mathrm{Fix}(x)$ and so $\mathrm{Fix}(x) = \mathrm{Fix}(x')$.
(d) Let $x' \in \mathcal{O}(x)$. By (a), $x \in \mathcal{O}(x')$, i.e., $t.x' = x$ for some $t \in T$. There is a $k \in K$ such that $t + k \in S$. Hence $x' = (t + k).x' = k.(t.x') = k.x$. Thus $\mathcal{O}(x) = K.x$ and, in particular, $\mathcal{O}(x)$ is a compact set.
(e) By (a) and (d), (T, X) is a compact minimal semiflow. $K + K$ is a compact subset of T and, since X is also compact, it acts uniformly equicontinuously on X. Fix an $\varepsilon > 0$. Let $\delta = \delta(\varepsilon, K + K) > 0$ be such that

$$(\forall\, x_1, x_2 \in X)\, (\forall\, r \in K + K)\; d(x_1, x_2) < \delta \Rightarrow d(r.x_1, r.x_2) < \varepsilon. \tag{1}$$

Let $x, y \in X$ and $t \in T$. There are $k \in K$ and $s \in S$ such that $t + k = s$. Since $X = K.(k.x)$, there is a $k' \in K$ such that $k'.(k.x) = t.x$. Hence $(2k + k').x = (t + k).x = x$ and so $2k + k' \in S$. Hence $(2k + k').(t.y) = t.y$, whence $k'.(k.y) = t.y$. If $d(x, y) < \varepsilon$, then, by (1), $d(t.x, t.y) = d((k' + k).x, (k' + k).y) < \delta$. Hence (T, X) is uniformly equicontinuous. □

Definition 2.4. A semiflow (T, X) is said to be *topologically transitive* (resp. *syndetically transitive*) if for any nonempty open subsets U, V of X the set $D(U, V) = \{t \in T \mid tU \cap V \neq \emptyset\}$ is nonempty (resp. syndetic).

Proposition 2.5. *([11, Theorem 3.3]) If the semiflow (T, X) is topologically transitive and the set of periodic points is dense in X, then (T, X) is syndetically transitive.*

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