Topology and its Applications 194 (2015) 296–305

Contents lists available at ScienceDirect

Topology and its Applications

www.elsevier.com/locate/topol

On scannable properties of the original knot from a knot shadow

Ryo Hanaki

Department of Mathematical Education, Nara University of Education, Takabatake, Nara, 630-8528, Japan

ARTICLE INFO

Article history:
Received 12 March 2015
Received in revised form 30 July 2015
Accepted 14 August 2015
Available online 2 September 2015

Keywords:
Knot shadow
Pseudo diagram
Trivializing number
Unknotting number
Genus
Positive knot

ABSTRACT

A knot shadow is a diagram with all crossing information missing. We cannot determine the original knot from a knot shadow in general. In this paper, we investigate properties (unknotting number, genus, braid index, etc.) of the original knot from a knot shadow.

1. Introduction

We consider oriented knots in $\mathbf{R}^3$ and do not distinguish between a knot and its knot type so long as no confusion occurs. For the standard definitions and results of knots and links, we refer to [2]. We say that a diagram of a knot K with all crossing information missing is a *knot shadow* S. Then, we call S a *knot shadow of* K and a crossing without crossing information a *precrossing*. We remark that a knot shadow is usually called a knot projection. Here, we consider a projection to be a shadow in this paper. A diagram D *is obtained from a knot shadow* S if D with all crossing information missing is S. We start from the following question.

Question 1. *Is the knot shadow S the projection of a diagram of the knot K, where S and K are pictured in Fig. 1?*

We can answer this question if we check all knots represented by diagrams obtained from S. However, it is difficult to check such all knots as the number of the precrossings of S increases. Now, therefore, we

E-mail address: hanaki@nara-edu.ac.jp.

http://dx.doi.org/10.1016/j.topol.2015.08.012
0166-8641/

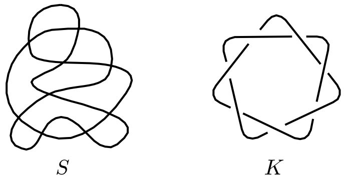

Fig. 1. Knot shadow.

investigate scannable properties (unknotting number, genus, braid index etc.) of the original knot from a knot shadow. We present an answer of this question in Section 3. We remark here that DNA knots originally inspired this research, namely we cannot determine over/under information at crossings in some photos of DNA knots.

For a knot shadow S we say that

$$\mathcal{K}_S = \{K \mid K \text{ has a knot shadow } S\}.$$

It is well-known that for any knot shadow S, $\mathcal{K}_S$ contains a trivial knot. We define the following:

$$u(S) = \max\{u(K) \mid K \in \mathcal{K}_S\},$$
$$c(S) = \max\{c(K) \mid K \in \mathcal{K}_S\},$$
$$g(S) = \max\{g(K) \mid K \in \mathcal{K}_S\},$$
$$br(S) = \max\{br(K) \mid K \in \mathcal{K}_S\},$$
$$b(S) = \max\{b(K) \mid K \in \mathcal{K}_S\},$$
$$\sigma(S) = \max\{|\sigma(K)| \mid K \in \mathcal{K}_S\},$$
$$s(S) = \max\{|s(K)| \mid K \in \mathcal{K}_S\}$$

where $u(K)$ is the unknotting number of K, $c(K)$ is the crossing number of K, $g(K)$ is the genus of K, $br(K)$ is the braid index of K, $b(K)$ is the bridge number of K, $\sigma(K)$ is the signature of K, and $s(K)$ means the Rasmussen invariant of K defined in [14]. Then we call $u(S)$ the *unknotting number of* S, $c(S)$ the *crossing number of* S, $g(S)$ the *genus of* S, $br(S)$ the *braid index of* S, $b(S)$ the *bridge number of* S, $\sigma(S)$ the *signature of* S, and $s(S)$ the *Rasmussen invariant of* S. Since the signature and the Rasmussen invariant of a knot take nonnegative integer, we take absolute values of them. Then, let $\overline{K}$ be the mirror image of a knot K,

$$\sigma(K) = -\sigma(\overline{K}), \quad s(K) = -s(\overline{K}).$$

Next, we define

$$u_{\min}(S) = \min\{u(K) \mid K \in \mathcal{K}_S\}.$$

Immediately, we see that for any knot shadow S, $u_{\min}(S) = 0$. Therefore, we do not define the minimum number of their invariants for knot shadows.

We investigate $u(S)$ in Section 3, $c(S)$ in Section 4.1, $g(S)$ in Section 4.2, and $br(S)$, $b(S)$, $\sigma(S)$ and $s(S)$ in Section 4.3.

We introduce related topics. We denote the set of all knot shadows of K by $\mathrm{PROJ}(K)$. A knot K_1 is a *minor* of K_2, denoted by $K_1 \leq K_2$ ($K_2 \geq K_1$), if $\mathrm{PROJ}(K_1) \supset \mathrm{PROJ}(K_2)$ in [16]. We denote the set of all knots by $\mathfrak{K}$. The following holds.