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Meshed continua have unique second and third symmetric products

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ABSTRACT

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1. Introduction

As we see in this section, the theory of uniqueness of hyperspaces is extensively researched, the *n*-th symmetric product is studied in this way. It has been proved by Hernández-Gutiérrez and Martínez-de-la-Vega [11] that wired continua have unique hyperspace $F_n(X)$, for $n \ge 4$. The authors of this paper prove that meshed continua have unique hyperspace $F_n(X)$, for each $n \in \{2, 3\}$.

A continuum is a nondegenerate, compact, connected metric space. The set of positive integers is denoted by N. For a given continuum X and $n \in \mathbb{N}$, we consider the following hyperspace of X,

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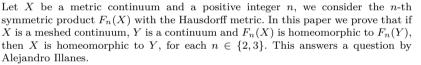
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 $2^{X} = \{A \subset X : A \text{ is a nonempty closed subset of } X\},$ $C_{n}(X) = \{A \in 2^{X} : A \text{ has at most } n \text{ components}\},$ $F_{n}(X) = \{A \in 2^{X} : A \text{ has at most } n \text{ points}\},$ $C(X) = C_{1}(X);$

all these hyperspaces are metrized by the Hausdorff metric (see [25, Definition 0.1]). The hyperspaces $F_n(X)$ and $C_n(X)$ are called the *n*-th symmetric product of X and the *n*-fold hyperspace of X, respectively.

Let \mathcal{K} be a class of continua, $n \in \mathbb{N}$ and $X \in \mathcal{K}$, we say that X has unique hyperspace $F_n(X)$ whenever Y is a continuum such that $F_n(X)$ is homeomorphic to $F_n(Y)$, it follows that X is homeomorphic to Y.

The topic of this paper is inserted in the following general problem.

Problem. Find conditions, on the continuum X, in order that X has unique hyperspace $F_n(X)$.

A finite graph is a continuum that can be written as the union of finitely many arcs, each two of which are either disjoint or intersect only in one or both of their end points. Let $\mathfrak{G} = \{X: X \text{ is a finite graph}\}$; it has been proved, for \mathfrak{G} , the following result.

(a) If $X \in \mathfrak{G}$ and $n \in \mathbb{N}$, then X has unique hyperspace $F_n(X)$ (see [6, Corollary 5.9]).

A *Peano continuum* is a locally connected continuum. Let $\mathfrak{LC} = \{X: X \text{ is a Peano continuum}\}$. A *dendrite* is a Peano continuum without simple closed curves. Let $\mathfrak{D} = \{X: X \text{ is a dendrite whose set of end points is closed}\}$. Notice that $\mathfrak{G} \not\subset \mathfrak{D}$ and $\mathfrak{D} \not\subset \mathfrak{G}$, it has been proved, for \mathfrak{D} , the following result.

(b) If $X \in \mathfrak{D}$ and $n \in \mathbb{N}$, then X has unique hyperspace $F_n(X)$ (see [2, Theorem 5.2], [15, Theorem 3.7]).

Let $\mathcal{O} = \{X: X \text{ is a dendrite whose set of ordinary points is open}\}$. Notice that $\mathfrak{D} \subsetneq \mathcal{O}$ (see [15, Corollary 2.4]).

(c) If $X, Y \in \mathcal{O}$ and $F_2(X)$ is homeomorphic to $F_2(Y)$, then X is homeomorphic to Y (see [19, Theorem 8]).

Given a continuum X, let

 $\mathcal{G}(X) = \{x \in X : x \text{ has a neighborhood } G \text{ in } X \text{ such that } G \text{ is a finite graph} \}$

and let

$$\mathcal{P}(X) = X - \mathcal{G}(X).$$

We recall that, as in [9], a continuum X is said to be almost meshed provided that the set $\mathcal{G}(X)$ is dense in X; and an almost meshed continuum X is meshed provided that X has a basis of neighborhoods \mathfrak{B} such that $U - \mathcal{P}(X)$ is connected, for each element $U \in \mathfrak{B}$. Let $\mathcal{AM} = \{X: X \text{ is an almost meshed continuum}\}$, and let $\mathcal{M} = \{X: X \text{ is a meshed continuum}\}$.

(d) If $X \in \mathcal{AM} \cap \mathfrak{LC}$ and $n \in \mathbb{N} - \{2, 3\}$, then X has unique hyperspace $F_n(X)$, [18, Corollary 4.4].

We recall that, as in [10], a *wire* in a continuum X is a subset α of X such that α is homeomorphic to one of the spaces (0, 1), [0, 1), [0, 1] or the unit circle S^1 in the Euclidean plane, and α is a component of

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