Topology and its Applications 191 (2015) 16–27

Contents lists available at ScienceDirect

Topology and its Applications

www.elsevier.com/locate/topol

Meshed continua have unique second and third symmetric products

Luis A. Guerrero-Méndez, David Herrera-Carrasco, María de J. López, Fernando Macías-Romero *

Facultad de Ciencias Físico Matemáticas de la Benemérita Universidad Autónoma de Puebla, Avenida San Claudio y 18 Sur, Colonia San Manuel, Edificio 111A-109B, Ciudad Universitaria, C.P. 72570 Puebla, Pue., Mexico

A R T I C L E I N F O

Article history:
Received 18 December 2013
Received in revised form 27 May 2014
Accepted 29 April 2015
Available online 21 May 2015

MSC:
54B20
54C50

Keywords:
Almost meshed
Continuum
Dendrite
Hyperspace
Local dendrite
Meshed
Peano continuum
Unique hyperspace

A B S T R A C T

Let X be a metric continuum and a positive integer n, we consider the n-th symmetric product $F_n(X)$ with the Hausdorff metric. In this paper we prove that if X is a meshed continuum, Y is a continuum and $F_n(X)$ is homeomorphic to $F_n(Y)$, then X is homeomorphic to Y, for each $n \in \{2, 3\}$. This answers a question by Alejandro Illanes.

© 2015 Elsevier B.V. All rights reserved.

1. Introduction

As we see in this section, the theory of uniqueness of hyperspaces is extensively researched, the n-th symmetric product is studied in this way. It has been proved by Hernández-Gutiérrez and Martínez-de-la-Vega [11] that wired continua have unique hyperspace $F_n(X)$, for $n \geq 4$. The authors of this paper prove that meshed continua have unique hyperspace $F_n(X)$, for each $n \in \{2, 3\}$.

A *continuum* is a nondegenerate, compact, connected metric space. The set of positive integers is denoted by $\mathbb{N}$. For a given continuum X and $n \in \mathbb{N}$, we consider the following hyperspace of X,

* Corresponding author.
E-mail addresses: luisgm@alumnos.fcfm.buap.mx (L.A. Guerrero-Méndez), dherrera@fcfm.buap.mx (D. Herrera-Carrasco), mjlopez@fcfm.buap.mx (M.d.J. López), fmacias@fcfm.buap.mx (F. Macías-Romero).

http://dx.doi.org/10.1016/j.topol.2015.04.018
0166-8641/© 2015 Elsevier B.V. All rights reserved.

$$2^X = \{A \subset X: A \text{ is a nonempty closed subset of } X\},$$
$$C_n(X) = \{A \in 2^X: A \text{ has at most } n \text{ components}\},$$
$$F_n(X) = \{A \in 2^X: A \text{ has at most } n \text{ points}\},$$
$$C(X) = C_1(X);$$

all these hyperspaces are metrized by the *Hausdorff metric* (see [25, Definition 0.1]). The hyperspaces $F_n(X)$ and $C_n(X)$ are called the *n-th symmetric product of* X and the *n-fold hyperspace* of X, respectively.

Let $\mathcal{K}$ be a class of continua, $n \in \mathbb{N}$ and $X \in \mathcal{K}$, we say that X has *unique hyperspace* $F_n(X)$ whenever Y is a continuum such that $F_n(X)$ is homeomorphic to $F_n(Y)$, it follows that X is homeomorphic to Y.

The topic of this paper is inserted in the following general problem.

Problem. Find conditions, on the continuum X, in order that X has unique hyperspace $F_n(X)$.

A *finite graph* is a continuum that can be written as the union of finitely many arcs, each two of which are either disjoint or intersect only in one or both of their end points. Let $\mathfrak{G} = \{X: X \text{ is a finite graph}\}$; it has been proved, for $\mathfrak{G}$, the following result.

(a) If $X \in \mathfrak{G}$ and $n \in \mathbb{N}$, then X has unique hyperspace $F_n(X)$ (see [6, Corollary 5.9]).

A *Peano continuum* is a locally connected continuum. Let $\mathfrak{LC} = \{X: X \text{ is a Peano continuum}\}$. A *dendrite* is a Peano continuum without simple closed curves. Let $\mathfrak{D} = \{X: X$ is a dendrite whose set of end points is closed$\}$. Notice that $\mathfrak{G} \not\subset \mathfrak{D}$ and $\mathfrak{D} \not\subset \mathfrak{G}$, it has been proved, for $\mathfrak{D}$, the following result.

(b) If $X \in \mathfrak{D}$ and $n \in \mathbb{N}$, then X has unique hyperspace $F_n(X)$ (see [2, Theorem 5.2], [15, Theorem 3.7]).

Let $\mathcal{O} = \{X: X$ is a dendrite whose set of ordinary points is open$\}$. Notice that $\mathfrak{D} \subsetneq \mathcal{O}$ (see [15, Corollary 2.4]).

(c) If $X, Y \in \mathcal{O}$ and $F_2(X)$ is homeomorphic to $F_2(Y)$, then X is homeomorphic to Y (see [19, Theorem 8]).

Given a continuum X, let

$$\mathcal{G}(X) = \{x \in X: x \text{ has a neighborhood } G \text{ in } X \text{ such that } G \text{ is a finite graph}\}$$

and let

$$\mathcal{P}(X) = X - \mathcal{G}(X).$$

We recall that, as in [9], a continuum X is said to be *almost meshed* provided that the set $\mathcal{G}(X)$ is dense in X; and an almost meshed continuum X is *meshed* provided that X has a basis of neighborhoods $\mathfrak{B}$ such that $U - \mathcal{P}(X)$ is connected, for each element $U \in \mathfrak{B}$. Let $\mathcal{AM} = \{X: X \text{ is an almost meshed continuum}\}$, and let $\mathcal{M} = \{X: X \text{ is a meshed continuum}\}$.

(d) If $X \in \mathcal{AM} \cap \mathfrak{LC}$ and $n \in \mathbb{N} - \{2, 3\}$, then X has unique hyperspace $F_n(X)$, [18, Corollary 4.4].

We recall that, as in [10], a *wire* in a continuum X is a subset α of X such that α is homeomorphic to one of the spaces $(0, 1)$, $[0, 1)$, $[0, 1]$ or the unit circle S^1 in the Euclidean plane, and α is a component of

Download English Version:

https://daneshyari.com/en/article/4658273

Download Persian Version:

https://daneshyari.com/article/4658273

Daneshyari.com