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Inverse limits with bonding functions whose graphs are arcs [☆]

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ABSTRACT

In this paper, we explore connectedness and total disconnectedness of generalized inverse limits on intervals induced by one bonding function $f : I \to 2^I$, which is not necessarily surjective, and whose graph $G(f)$ is an arc.

As the main results of the paper we prove that (1) if such inverse limit consists of more than one point, then it is infinite, and (2) if such a function f is surjective, then the corresponding inverse limit is never totally disconnected. As a by-product, we obtain a more general result: if for each i, f_i denotes a surjective upper semicontinuous bonding function with connected graph, then the resulting generalized inverse limit is never totally disconnected.

We also produce examples of such inverse limits (with nonsurjective bonding function f) that are totally disconnected.

1. Introduction

Continua as inverse limits have been studied intensively in last decades. One reason for such intense research is the fact that inverse sequences with very simple spaces and simple bonding maps can produce very complicated continua as their inverse limits. This may happen even in the case when all the spaces are closed unit intervals and all the bonding functions are the same. Such inverse limits also appear in such diverse areas as economy, mechanics of fluids, physics and more; for examples see [15–18,20]. There are also many examples of applications where models reduce to upper semicontinuous set-valued functions (i.e. the

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Christiano–Harrison model is a model from macroeconomics that reduces to a set-valued function) and therefore the concept of inverse limits of inverse sequences with upper semicontinuous set-valued bonding functions (or simply generalized inverse limits) is needed. Such a generalization of the concept of inverse limits was introduced in [14,19] by W.T. Ingram and W.S. Mahavier. The concept of these generalized inverse limits has become very popular since their introduction and has been studied by many authors and many papers have appeared; for examples see [1–4,8,11,14,19,22,24,26], where more references can be found.

In this paper, we explore properties of generalized inverse limits on intervals induced by one bonding function $f : I \to 2^I$,

1. which is not necessarily surjective, and
2. whose graph $G(f)$ is an arc.

Such inverse limits can be many things, from the usual inverse limits induced by a single bonding map $f : I \to I$ to one point (if one takes the inverse limit induced by the bonding map $f_a : I \to \{a\}$ for some a in I the result is an inverse limit consisting of only the point $\{(a, a, a, \ldots)\}$).

The usual inverse limits induced by a single bonding map $f : I \to I$ are always connected. This may not be true if the bonding function is not single-valued, even though the graph $G(f)$ is an arc [14].

In the present paper we explore connectedness and total disconnectedness of such generalized inverse limits. The research of connectedness of generalized inverse limits has proved to be very challenging. It has been studied very intensively and many papers have appeared, i.e. [4,6,5,10,12–14,24].

In this paper we present examples of such inverse limits (with nonsurjective bonding function f) that are totally disconnected, i.e. they can either be

1. degenerate (one-point continua),
2. countable, or
3. uncountable (they may also be homeomorphic to the Cantor set).

As the main results of the paper we prove that

1. if such an inverse limit consists of more than one point, then it is infinite, and
2. if such a function f is surjective, then the corresponding inverse limit is never totally disconnected.

As a by-product, we obtain a more general result: if for each i, f_i denotes a surjective upper semicontinuous bonding function with connected graph, then the resulting generalized inverse limit is never totally disconnected.

We conclude the paper by introducing an interesting open problem.

The original version of reference [4] (Generic generalized inverse limits) contained a significant error. The authors discovered the error and retracted the paper a year before it was published by mistake. (The intersection of the set of authors of that paper and the authors of this paper is the author who made the error—that intersection being the set whose only element is Judy Kennedy.) Anyway, the astute reader may notice that the results in the original, erroneously published paper and this paper (in particular, Theorem 4.5 of this paper) cannot both be true and they would be correct. In the original "Generic generalized inverse limits" paper, it was claimed that generic generalized inverse limits on intervals are totally disconnected. This is not true, as Theorem 4.5 of this paper demonstrates. It is true that generic generalized inverse limits on intervals are disconnected, and, in fact, that, generically, all sub-generalized inverse limits of a generalized inverse limit space on intervals are disconnected. This is proven in the corrected version "Generic generalized inverse limits: corrections and extensions".