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On the number of orbits of the homeomorphism group of solenoidal spaces

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ABSTRACT

A continuum X is called solenoidal if it is circle-like and nonplanar. X is $\frac{1}{n}$-homogeneous if the action of its homeomorphism group Homeo(X) on X has exactly n orbits; i.e. there are exactly n types of points in X. Recently Jiménez-Hernández, Minc and Pellicer-Covarrubias [6] constructed a family of $\frac{1}{n}$-homogeneous solenoidal continua, for every $n > 2$. Modifying the spaces obtained by them, as well as an earlier construction of the author for $n = 2$, for every $n > 2$ we construct two different uncountable families of arcless $\frac{1}{n}$-homogeneous solenoidal continua Σ_n and Σ'_n. We also show that there is an uncountable family of countably nonhomogeneous solenoidal continua Σ_∞; i.e. each $Y \in \Sigma_\infty$ has (infinitely) countably many types of points. For every $Y \in \bigcup_{n\in\mathbb{N}} \Sigma_n \cup \Sigma_\infty$ any orbit of Homeo(Y) is uncountable. With respect to the degree of homogeneity, in the realm of solenoidal continua containing pseudoarcs, our examples complete the gap between homogeneous solenoids of pseudoarcs and uncountably nonhomogeneous pseudosolenoids. A number of questions related to the study is raised.

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1. Introduction

A *continuum* is a compact and connected metric space that contains at least two points. A continuum is *arcless* if it contains no arcs. A continuum X is called *solenoidal* if it is circle-like and nonplanar. It is well known that all solenoidal continua are indecomposable (see [17, Theorems 2 & 3]). For a continuum Y by Homeo(Y) we shall denote its *homeomorphism group*. We say that Y is $\frac{1}{n}$*-homogeneous* if the action of Homeo(Y) on Y has exactly n orbits. If the number of such orbits is countable (resp. uncountable), we say that Y is *countably* (resp. *uncountably*) *nonhomogeneous*. Recently there has been an increased interest in the degree of homogeneity of continua and its relation to other topological properties (see e.g. [3,6,9–16, 20]). One of such properties, whose connection to the degree of homogeneity is still not well understood, is

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indecomposability in dimension one. In fact still only a very limited number of examples of $\frac{1}{n}$-homogeneous indecomposable one-dimensional continua can be found in the literature. Answering a question raised in [13] the author constructed in [4] a $\frac{1}{2}$-homogeneous solenoidal continuum. A topologically equivalent example was independently obtained in [16] by P. Pyrih and B. Vejnar. In [6] R. Jiménez-Hernández, P. Minc and P. Pellicer-Covarrubias constructed a family of $\frac{1}{n}$-homogeneous solenoidal continua for every $n > 2$. Their spaces were obtained as the inverse limit of circle-like planar continua, each of which consisted of various modifications of the $\sin(1/x)$-continuum. Consequently the solenoidal spaces constructed by them, each have the property that any of their proper subcontinua contain an arc. In the present paper we shall construct two new distinct uncountable families of $\frac{1}{n}$-homogeneous arcless solenoidal continua for every $n > 2$. The first of them is easily obtained by the following result of W. Lewis [8], application of which, for a fixed space S_n constructed in [6], gives a new space $\bar{S}_n$ that could be called "S_n of pseudoarcs". A detailed description is given in Section 4.

Theorem A. *(Lewis [8]) For every 1-dimensional continuum M there exists a 1-dimensional continuum $\hat{M}$ that has a continuous decomposition into pseudoarcs such that the decomposition space is homeomorphic to M. Furthermore, every homeomorphism of M onto itself can be lifted to a homeomorphism of $\hat{M}$, with a free motion within the decomposition elements.*

The second family is based on a solenoid of pseudoarcs with sequences of pseudoarcs pinched to sequences of points. Any proper subcontinuum of any element in the family is either a pseudoarc, an arc of pseudoarcs or the latter with some of the pseudoarcs pinched to points. Our construction is a modification of the $\frac{1}{2}$-homogeneous example given by the author in [4]. Let us outline the major steps of this construction, starting with a summary of the case $n = 2$ from [4].

- Let X be the circle of pseudoarcs with a continuous decomposition $\{P_x : x \in \mathbb{S}^1\}$.
- Fix $c \in \mathbb{S}^1$ and pinch the pseudoarc P_c to a point to obtain a (decomposable) $\frac{1}{2}$-homogeneous circle-like planar continuum X_1.
- For every m there is a $\frac{1}{2}$-homogeneous circle-like planar continuum X_m that 2^{m-1}-fold covers X_1 by $\tau_m : X_{m+1} \to X_m$.
- The inverse limit $S_X = \lim_{\leftarrow}\{X_m, \tau_m : m \in \mathbb{N}\}$ is an indecomposable $\frac{1}{2}$-homogeneous circle-like continuum.

A useful property of X_1 is that one of the two orbits of $\mathrm{Homeo}(X_1)$ consists of a single point that locally separates X_1. The idea behind our construction of a $\frac{1}{n}$-homogeneous solenoidal continuum is to modify X_1 in such a way so that the kth Cantor–Bendixson derivative of the set L of points that locally separate X_1 and are isolated in L is nonempty, if and only if $k \leq n - 2$. This is obtained by induction, starting with a $\frac{1}{3}$-homogeneous circle-like planar continuum obtained from X_1 by introducing a sequence of local separating points convergent to a local separating point. Following the earlier approach of the author it will be convenient to carry out the entire construction in terms of covering spaces, as well as one-point and two-point compactifications of the universal covers. The advantage of this approach is that one can then use lifting properties of annulus maps and this way easily describe the desired elements of $\mathrm{Homeo}(X)$. The detailed description of the entire construction can be found in Section 3.

2. Preliminaries

Suppose X and Y are two topological spaces. We shall write $X \cong Y$ to denote the fact that X and Y are homeomorphic. Given a sequence of continua $\{X_i : i \in \mathbb{N}\}$ and *bonding maps* $\{f_i : X_{i+1} \to X_i | i \in \mathbb{N}\}$ their *inverse limit space* is given by

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