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Seifert surgery on knots via Reidemeister torsion and Casson–Walker–Lescop invariant

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ABSTRACT

For a knot K with $\Delta_K(t) \doteq t^2 - 3t + 1$ in a homology 3-sphere, let M be the result of $2/q$-surgery on K. We show that appropriate assumptions on the Reidemeister torsion and the Casson–Walker–Lescop invariant of the universal abelian covering of M imply $q = \pm 1$, if M is a Seifert fibered space.

1. Introduction

Dehn surgeries on knots or links have been studied from various points of view (e.g. [1–8,12,13,16,17,22–25]). The first author [5] introduced an idea for applying the Reidemeister torsion to Dehn surgery, and showed the following:

Theorem 1.1. *([6, Theorem 1.4]) Let K be a knot in a homology 3-sphere Σ such that the Alexander polynomial of K is $t^2 - 3t + 1$. The only surgeries on K that may produce a Seifert fibered space with base S^2 and with $H_1 \neq \{0\}, \mathbb{Z}$ have coefficients $2/q$ and $3/q$, and produce Seifert fibered space with three singular*

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fibers. Moreover (1) if the coefficient is $2/q$*, then the set of multiplicities is* $\{2\alpha, 2\beta, 5\}$ *where* $\gcd(\alpha, \beta) = 1$*, and (2) if the coefficient is* $3/q$*, then the set of multiplicities is* $\{3\alpha, 3\beta, 4\}$ *where* $\gcd(\alpha, \beta) = 1$.

In this paper, based on Theorem 1.1, we discuss the $2/q$ – Seifert surgery by applying the Reidemeister torsion and the Casson–Walker–Lescop invariant in combination simultaneously, and give a sufficient condition to determine the integrality of $2/q$ (Theorem 2.1). The condition is suggested by computations for the figure eight knot (Example 2.2).

This paper is actually a continuation of [6], so we follow mainly the notations of [6] and review necessary minimum ones:

(1) Let Σ be a homology 3-sphere, and let K be a knot in Σ. Then $\Delta_K(t)$ denotes the Alexander polynomial of K, and $\Sigma(K; p/r)$ denotes the result of p/r-surgery on K.
(2) Let ζ_d be a primitive d-th root of unity. For an element α of $\mathbb{Q}(\zeta_d)$, $N_d(\alpha)$ denotes the norm of α associated to the algebraic extension $\mathbb{Q}(\zeta_d)$ over $\mathbb{Q}$. Let $f(t)$ be a Laurent polynomial over $\mathbb{Z}$. We define $|f(t)|_d$ by

$$|f(t)|_d = |N_d(f(\zeta_d))| = \left| \prod_{i \in (\mathbb{Z}/d\mathbb{Z})^\times} f(\zeta_d^i) \right|.$$

Let X be a homology lens space with $H_1(X) \cong \mathbb{Z}/p\mathbb{Z}$. Let d be a divisor of p. We define $|X|_d$ by

$$|X|_d = |\Delta_K(t)|_d,$$

where K is a knot in a homology 3-sphere Σ such that $X = \Sigma(K; p/r)$. Then $|X|_d$ is a topological invariant of X (Refer to [7] for details).
(3) Let X be a closed oriented 3-manifold. Then $\lambda(X)$ denotes the Lescop invariant of X ([10]). Note that $\lambda\left(S^3\right) = 0$.

2. Result

Let K be a knot in a homology 3-sphere Σ. Let M be the result of $2/q$-surgery on K: $M = \Sigma(K; 2/q)$. Let $\pi : X \to M$ be the universal abelian covering of M (i.e. the covering associated to $\mathrm{Ker}(\pi_1(M) \to H_1(M))$). Since $H_1(M) \cong \mathbb{Z}/2\mathbb{Z}$, π is the 2-fold unbranched covering.

We then define $\lambda_q(K)$ by the following formula:

$$\lambda_q(K) := \lambda(X).$$

It is obvious that $\lambda_q(K)$ is a knot invariant of K. We also define $|K|_{(q,d)}$ by the following formula, if $|X|_d$ is defined:

$$|K|_{(q,d)} := |X|_d.$$

It is also obvious that $|K|_{(q,d)}$ is a knot invariant of K.

We then have the following.

Theorem 2.1. *Let* K *be a knot in a homology 3-sphere* Σ*. We assume the following.*

(2.1) $\lambda(\Sigma) = 0$,
(2.2) $\Delta_K(t) \doteq t^2 - 3t + 1$,