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Cardinal invariants in locally T_i-minimal paratopological groups ☆

Jing Zhang [a,*], Wei He [b]

[a] School of Mathematics and Statistics, Minnan Normal University, Zhangzhou 363000, PR China
[b] Institute of Mathematics, Nanjing Normal University, Nanjing 210046, PR China

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ABSTRACT

In this paper, cardinal invariants in locally T_i-minimal paratopological groups with $i = 1, 2, 3$ are studied. It mainly shows that: (1) If (G, τ) is a T_2 locally T_1-minimal 2-oscillating paratopological group, then $\chi(G) = \pi\chi(G) \cdot inv(G)$; (2) Let (G, τ) be a locally T_1-minimal paratopological group, then $\chi(G) = \psi(G) \cdot inv(G)$; (3) If (G, τ) is a locally T_2-minimal paratopological group, then $\chi(G) = \psi(G) \cdot inv(G) \cdot Hs(G)$; (4) If (G, τ) is a locally T_3-minimal paratopological group, then $\chi(G) = \psi(G) \cdot inv(G) \cdot Ir(G)$. These results generalize the corresponding results in [9] and also give positive answers to two questions posed by F.C. Lin in [9].

1. Introduction

In this paper, all spaces are assumed to be T_0 unless stated otherwise. Moreover, we assume that T_3 spaces are T_1. The notations ω, $\psi(G)$ and $\chi(G)$ mark the set of all non-negative integers, the pseudocharacter and character of a space G, respectively. We denote by $\mathbb{N}$ the set of natural numbers. The letter e will always denote the neutral element of a paratopological group. The readers may refer to [2,7] for notations and terminologies not explicitly given here.

Recall that a paratopological group is a group G with a topology such that the multiplication in G is jointly continuous. A paratopological group with a continuous inverse mapping is called a topological group.

A Hausdorff topological group (G, τ) is called minimal if there is no Hausdorff group topology on G which is strictly coarser than τ. Minimal topological groups have been studied in detail in [6,15]. One of the

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* Corresponding author.
E-mail addresses: zhangjing86@126.com (J. Zhang), weihe@njnu.edu.cn (W. He).

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generalizations of minimality of topological group is local minimality which was introduced by Morris and Pestov in [12]. A Hausdorff topological group (G, τ) is called locally minimal if there exists a neighborhood U of the neutral element e in (G, τ) such that U fails to be a neighborhood of e in any Hausdorff group topology on G which is strictly coarser than τ. For recent progress in this field see [3,4].

It is well known that paratopological groups are good generalizations of the category of topological groups. In the past few years, paratopological groups have been largely studied, for example, in [1,2,5,11]. Motivated by the idea of the (locally) minimal topological groups, I. Guan [8] defined the concept of the minimal Hausdorff paratopological groups and F.C. Lin introduced locally T_i-minimal paratopological groups with $i = 0, 1, 2, 3, 3.5$. Usually, no implication $T_i \Rightarrow T_j$ for $i < j$ is valid in paratopological groups. This is why F.C. Lin introduced the concept of locally T_i-minimal paratopological groups. For $i = 0, 1, 2, 3, 3.5$, a T_i paratopological group (G, τ) is called locally T_i-minimal [9] if there exists a τ-neighborhood V of the neutral element e such that whenever $\sigma \leq \tau$ is a T_i-paratopological group topology on G such that V is a σ-neighborhood of e, then $\sigma = \tau$. A paratopological group is called locally minimal [9] if it is a locally T_i-minimal paratopological group for each $i = 0, 1, 2, 3, 3.5$.

Cardinal functions are interesting topics in the category of general topology. Many topologists have investigated cardinal invariants in topological groups and paratopological groups extensively. In [9], the author proved the following results:

Theorem 1.1. *If (G, τ) is a regular locally T_1-minimal Abelian paratopological group, then $\chi(G) = \psi(G)$.*

Theorem 1.2. *If (G, τ) is an Abelian locally T_3-minimal paratopological group, then $\omega(G) = n\omega(G)$.*

The following questions are posed in [9]:

Question 1.1. Is every countable locally T_3-minimal paratopological group metrizable?

Question 1.2. Let G be a locally T_i-minimal paratopological group for $i = 0$, or $i = 1$, or $i = 2$, or $i = 3$, or $i = 3.5$. Does one have $n\omega(G) = \omega(G)$?

In this paper we will investigate the cardinal invariants in locally T_i-minimal paratopological groups with $i = 1, 2, 3$, in particular, we will give positive answers to the above Questions 1.1 and 1.2.

2. Preliminaries

T. Banakh and O. Ravsky in [5] introduce oscillator topology on a paratopological group. Given a paratopological topological group G let $\tau_\flat$ be the strongest group topology on G, weaker than the topology of G. Given a subset U of a group G, define the sets $(\pm U)^n$ and $(\mp U)^n$ by letting $(\pm U)^n = UU^{-1}U \cdots U^{(-1)^{n-1}}$, $(\mp U)^n = U^{-1}UU^{-1} \cdots U^{(-1)^n}$ for $n \in \omega$ and $(\pm U)^0 = (\mp U)^0 = \{e\}$. An *n-oscillator* on a paratopological group (G, τ) is a set of the form $(\pm U)^n$ for some neighborhood U of e in G.

For *n-oscillator topology* on a paratopological group (G, τ) we mean the topological space (G, τ_n) with τ_n consisting of sets $U \subset G$ such that for each $x \in U$ there is an n-oscillator $(\pm V)^n$ with $x \cdot (\pm V)^n \subset U$. In general, (G, τ_n) is not a paratopological group but it is a semitopological group, that is, τ_n makes the group operation on G separately continuous.

A paratopological group G has finite oscillation if there exists $n \in \mathbb{N}$ such that (G, τ_n) is a topological group. Let $\mathrm{osc}(G)$ be the smallest positive n such that (G, τ_n) is a topological group. We shall say that a paratopological group G is n-oscillating if $\mathrm{osc}(G) \leq n$. In particular a 2-oscillating paratopological group means that the sets UU^{-1} where U is an open neighborhood of e in G form a neighborhood base at e in $(G, \tau_\flat)$.