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On instanton homology of corks W_n

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ABSTRACT

We consider a family of corks, denoted W_n, constructed by Akbulut and Yasui. Each cork gives rise to an exotic structure on a smooth 4-manifold via a twist τ on its boundary $\Sigma_n = \partial W_n$. We compute the instanton Floer homology of Σ_n and show that the map induced on the instanton Floer homology by $\tau : \Sigma_n \to \Sigma_n$ is non-trivial.

1. Introduction

In [1], Akbulut and Yasui defined a cork C as a compact Stein 4-manifold with boundary together with an involution $\tau : \partial C \to \partial C$ which extends as a self-homeomorphism of C but not as a self-diffeomorphism. In addition, $C \subset X$ is a cork of a smooth 4-manifold X if cutting C out and regluing it via τ changes the diffeomorphism type of X.

We will consider the family of corks W_n, $n \geqslant 1$, obtained by surgery on the link in Fig. 1 where a positive integer m in a box indicates m right-handed half-twists. The boundary Σ_n of W_n is the integral homology 3-sphere with surgery description as in Fig. 2. The involution $\tau : \Sigma_n \to \Sigma_n$ interchanges the two components of the link in Fig. 2. It is best seen when the underlying link L_n is drawn symmetrically, as in Fig. 3. Note that Fig. 3 is obtained from Fig. 2 by an isotopy that drags the lower arc of the left component underneath the lower arc of the right component.

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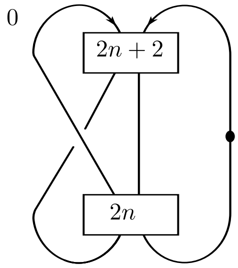

Fig. 1. W_n.

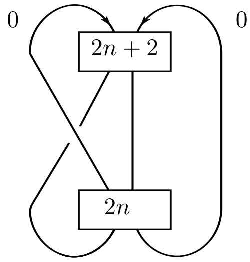

Fig. 2. Σ_n.

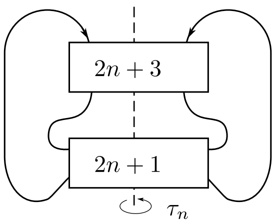

Fig. 3. L_n.

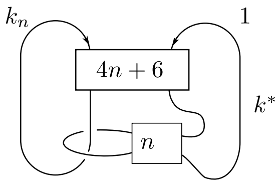

Fig. 4. S^3.

The quotient manifold $\Sigma'_n = \Sigma_n/\tau$ is obtained from 1-surgery on the knot k^* in Fig. 4. Note that the preimage of k^* under projection is the link L_n. The canonical longitude of each component of L_n projects onto a longitude of the knot k^* that links k^* with linking number equal to one. Since k^* is an unknot, we have that Σ'_n is homeomorphic to S^3 and therefore Σ_n can be viewed as a double branched cover of S^3 with branch set k_n as shown in Fig. 4.

The goal of this paper is to study the instanton Floer homology $I_*(\Sigma_n)$ and the map $\tau_* : I_*(\Sigma_n) \to I_*(\Sigma_n)$ induced on it by τ.

Theorem 1.

(1) *For every integer $n \geqslant 1$, the instanton Floer homology group $I_j(\Sigma_n)$, $j = 0, \dots, 7$, is trivial if j is even, and is a free abelian group of rank $n(n+1)(n+2)/6$ if j is odd.*
(2) *The homomorphism $\tau_* : I_*(\Sigma_n) \to I_*(\Sigma_n)$ is non-trivial for all $n \geqslant 1$.*

The first example of an involution acting non-trivially on the instanton Floer homology of an irreducible homology 3-sphere was given in [2] and [3]; in fact, that example was exactly our $\tau : \Sigma_1 \to \Sigma_1$. The technique we use to show non-triviality of τ_* is the same as the technique that was used in [4] to reprove the result of [3]: compare the Lefschetz number of $\tau_* : I_*(\Sigma_n) \to I_*(\Sigma_n)$ with the Lefschetz number of the identity map. If the two are different, then the involution must be non-trivial. For any integral homology 3-sphere Σ, the Lefschetz number of the identity equals the Euler characteristic of $I_*(\Sigma)$, which by Taubes [5] is twice the Casson invariant $\lambda(\Sigma)$. Ruberman and Saveliev [6] showed that the Lefschetz number of τ_* equals twice the equivariant Casson invariant $\lambda^\tau(\Sigma)$, defined in [7]. Therefore the non-triviality of τ_* will follow as soon as we show that $\lambda(\Sigma_n) \neq \lambda^\tau(\Sigma_n)$. The calculation of $I_*(\Sigma_n)$ is done using surgery techniques.

It should be noted that Akbulut and Karakurt proved a Heegaard Floer analogue of this result. In [8] they showed that the involution $\tau : \Sigma_n \to \Sigma_n$ acts non-trivially on the Heegaard Floer homology group $HF^+(\Sigma_n)$.