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An automatic step-size adjustment algorithm for LMS adaptive filters, and an application to channel estimation

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ABSTRACT

We propose a least-mean-square adaptive filter with automatic step-size adjustment (ASSA). At each time instant when a new observation of the input signal arrives, a new step-size parameter is chosen such that the sum of the squares of the measured estimation errors up to that current time instant is minimized. This step size, after being normalized by the power of the current tapped filter input, is used to update the filter weights for the next time instant. The filter weights are thus updated automatically without the aid of any preset control parameters. When applied to channel estimation, simulation results show the performance advantage of the ASSA algorithm over the existing step-size adjustment algorithms under different wireless channel environments.

1. Introduction

The least-mean-square (LMS) algorithm [1] is widely used in adaptive signal processing for its robustness and simplicity. To avoid the gradient noise amplification problem in the LMS algorithm, the normalized LMS (NLMS) algorithm [2] uses a step size that is inversely proportional to the signal level at the filter input. The step size is an important parameter that controls the trade-off between the rate of convergence and the steady-state mean-square error (MSE) of an algorithm. As is well-known, a variable step size is superior to a fixed step size as the former can respond to a changing environment and more importantly, it can provide a fast rate of convergence at the beginning of the adaptation process and arrive at a small steady-state MSE when the adaptive algorithm converges [3–6].

Consider in general an N-tap filter, with the weight vector $\mathbf{w}(n)$ at time instant n denoted by

$$\mathbf{w}(n) = [w_1(n)\ w_2(n)\ \cdots\ w_N(n)]^T. \tag{1}$$

Here, superscript T denotes transposition, and a boldface character represents a vector. Let $\{x(n)\}$ be the input sequence and $\mathbf{x}(n) = [x(n)\ x(n-1)\ \cdots\ x(n-N+1)]^T$ be its vector representation containing the immediate past N samples of $\{x(n)\}$. The filter output $y(n) = \mathbf{w}^T(n)\mathbf{x}(n)$ aims to follow a desired signal $d(n)$, and the estimation error $e(n)$ is defined as

$$e(n) = d(n) - y(n). \tag{2}$$

An adaptive filtering algorithm adjusts the filter tap-weight $\mathbf{w}(n)$ at each time instant according to the measured value of $e(n)$. Specifically, the standard LMS algorithm updates $\mathbf{w}(n)$ as [2]

$$\mathbf{w}(n+1) = \mathbf{w}(n) + \mu e(n)\mathbf{x}(n), \tag{3}$$

where μ is defined as the step-size parameter that affects the convergence behavior of the filter weights. The NLMS algorithm can be viewed as a special case of the LMS algorithm with a time-varying step size, in which the step size varies with the input signal strength. The tap-weight adaptation equation of the NLMS algorithm is given by [2]

$$\mathbf{w}(n+1) = \mathbf{w}(n) + \frac{\hat{\mu}}{\mathbf{x}^T(n)\mathbf{x}(n)} e(n)\mathbf{x}(n), \tag{4}$$

where $\hat{\mu}$ is a parameter to be chosen and $\hat{\mu}/\mathbf{x}^T(n)\mathbf{x}(n)$ is the actual step size. A more reliable implementation of the NLMS algorithm in practice requires the assistance of

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a regularization parameter ε, and it results in the ε-NLMS algorithm [2] with the improved step size $\hat{\mu}/(\mathbf{x}^T(n)\mathbf{x}(n)+\varepsilon)$. Although having the tap-weight adaptation equations in a similar form (and differing only in the choice of the step size), LMS-type algorithms are usually not compared with NLMS-type algorithms. In the literature, it has been shown that NLMS-type algorithms provide a potentially faster rate of convergence [2,7]. We thus focus on NLMS-type algorithms.

Following a procedure similar to that in [4,8] applies a normalized gradient adaptive descent algorithm to the $\hat{\mu}$ in (4), and arrives at the variable step-size NLMS (VSS-NLMS) algorithm. In [8], no regularization is performed, i.e., $\varepsilon = 0$, and it is noted that an initial value of $\hat{\mu}$ and an additional constant ρ are to be set for the algorithm to work. Instead of adjusting $\hat{\mu}$, [9] proposes to update ε according to a generalized normalized gradient descent algorithm. In [9], a parameter ρ controls the learning rate of the algorithm, and $\rho = 0.15$ is used as a typical value in the simulations. Ref. [10] proposes a robust regularization method where a sign function is applied to the gradient descent term derived in [9], and by using the typical setting of $\rho = 0.15$, it shows an improvement over the method of [9] in terms of the steady-state MSE. As can be seen, a common feature of the step-size adjustment methods above is that preset control parameters are required. These parameters are always chosen from extensive simulations, or from experience. It is obvious that the choice of parameters would greatly affect the performance of these algorithms. To obviate the tedious process of choosing appropriate parameters, parameter-free algorithms are desired. Ref. [11] presents a nonparametric variable step-size NLMS (NPVSS-NLMS) algorithm which aims to reduce the impact of the noise present in the desired response $d(n)$. In its case, $d(n)$ is corrupted by a system noise. When setting the power of this noise to zero, the NPVSS-NLMS algorithm reduces to the conventional NLMS algorithm. To the best of our knowledge, no parameter-free algorithms in the true sense have been introduced so far.

This paper presents the complete theoretical development for a control parameter-free algorithm for the tap-weight adaptation of an LMS adaptive filter that we call the automatic step-size adjustment (ASSA) algorithm [12], and provides extensive simulation results to demonstrate the superior performance of the ASSA algorithm over other existing algorithms that we mentioned above. At each time instant when a new observation of the input signal arrives, we choose a step size that would minimize the sum of the squares of the measured estimation errors up to that current time instant. This desired step size is, of course, only obtainable through genie-aided feedback. For a real system, we propose a suboptimum realization of the desired step size, where the past measured values of the estimation errors and the past realizations of the tap-weight vector are used. The obtained suboptimum step size, after being normalized by the power of the current tapped filter input, is used as the actual step size to update the current tap-weight vector. The adaptive filter employing this proposed algorithm can update its tap-weight vector online without setting any control parameters in advance. As an application to wireless communications, we apply the adaptive filter to channel estimation and examine the MSE performance of the ASSA algorithm using simulations. Simulation results show the effectiveness of the new algorithm in rapidly driving the MSE to a smaller value compared with other existing adaptive filtering algorithms. The ASSA algorithm is formulated in detail in Section 2 and its evaluation through simulations is presented in Section 3. Finally, Section 4 draws conclusions.

2. The ASSA algorithm

We begin the formulation by studying the first few rounds of the adaptation process, and then we generalize it to an online algorithm. Without loss of generality, we assume that the adaptation starts at time instant 0, where the input vector is $\mathbf{x}(0) = [x(0)\ x(-1)\ \cdots\ x(-N+1)]^T$, and the weight vector $\mathbf{w}(0)$ is chosen to be the zero vector. The estimation error is given by $e(0) = d(0) - \mathbf{w}^T(0)\mathbf{x}(0)$, and it is noted that $e(0) = d(0)$. At this stage, an arbitrarily chosen initial step size μ_0 can be used to update the weight vector as $\mathbf{w}(1) = \mathbf{w}(0) + \mu_0 e(0)\mathbf{x}(0)$, as long as μ_0 keeps $\mathbf{w}(1)$ stable, i.e., the *mis-adjustment* [2] by applying $\mathbf{w}(1)$ is finite. At time instant 1, a new observation $x(1)$ arrives and the input vector is now $\mathbf{x}(1) = [x(1)\ x(0)\ \cdots\ x(-N+2)]^T$. The estimation error is

$$\begin{aligned} e(1) &= d(1) - \mathbf{w}^T(1)\mathbf{x}(1) \\ &= d(1) - [\mathbf{w}(0) + \mu_0 e(0)\mathbf{x}(0)]^T\mathbf{x}(1). \end{aligned} \tag{5}$$

From the viewpoint of time instant 1, looking backwards, we propose that the step size would have been more suitably chosen at time instant 0 if $e^2(1)$ is minimized. Suppose this more suitable step size is $\tilde{\mu}_1$. If genie-aided feedback enables us to reset the step size for time instant 0 as $\tilde{\mu}_1$, we would have

$$\tilde{\mathbf{w}}_1(1) = \mathbf{w}(0) + \tilde{\mu}_1 e(0)\mathbf{x}(0) \tag{6}$$

$$\begin{aligned} \tilde{e}_1(1) &= d(1) - \tilde{\mathbf{w}}_1^T(1)\mathbf{x}(1) \\ &= d(1) - [\mathbf{w}(0) + \tilde{\mu}_1 e(0)\mathbf{x}(0)]^T\mathbf{x}(1). \end{aligned} \tag{7}$$

from which $\tilde{\mu}_1$ can be analytically determined. Now we take $\mu_1 = \tilde{\mu}_1/\mathbf{x}^T(1)\mathbf{x}(1)$ to be the actual step size used for updating the current tap-weight vector, i.e.

$$\mathbf{w}(2) = \tilde{\mathbf{w}}_1(1) + \mu_1\tilde{e}_1(1)\mathbf{x}(1). \tag{8}$$

Then, at time instant 2, with input vector $\mathbf{x}(2) = [x(2)\ x(1)\ \cdots\ x(-N+3)]^T$, the estimation error is given by

$$\begin{aligned} e(2) &= d(2) - \mathbf{w}^T(2)\mathbf{x}(2) \\ &= d(2) - [\tilde{\mathbf{w}}_1(1) + \mu_1\tilde{e}_1(1)\mathbf{x}(1)]^T\mathbf{x}(2). \end{aligned} \tag{9}$$

Now, again from the viewpoint of time instant 2, looking backwards, the step size would have been more suitably chosen for both time instants 0 and 1 at the same time, if $e^2(1)+e^2(2)$ is minimized. Suppose this more suitable step size is $\tilde{\mu}_2$. If genie-aided feedback allows us to reset the step size for both time instants 0 and 1 as $\tilde{\mu}_2$, we would have

$$\tilde{\mathbf{w}}_2(1) = \mathbf{w}(0) + \tilde{\mu}_2 e(0)\mathbf{x}(0) \tag{10}$$

$$\begin{aligned} \tilde{e}_2(1) &= d(1) - \tilde{\mathbf{w}}_2^T(1)\mathbf{x}(1) \\ &= d(1) - [\mathbf{w}(0) + \tilde{\mu}_2 e(0)\mathbf{x}(0)]^T\mathbf{x}(1), \end{aligned} \tag{11}$$