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Joint channel and Frequency Offset estimation in MIMO-OFDM systems with insufficient Cyclic Prefix☆

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ABSTRACT

A Maximum Likelihood joint channel and Frequency Offset estimator is addressed for Multiple-Input Multiple-Output Orthogonal Frequency Division Multiplexing systems in the presence of Inter Symbol Interference and Inter Carrier Interference due to an insufficient Cyclic Prefix. The use of known Pseudo Noise based Symmetric Sequences to perform the joint estimation is proposed. The periodicity of the cost function for the Frequency Offset estimator has been analytically derived. It has been found that the proposed Pseudo Noise based Symmetric Sequences achieve an acquisition range for normalized Frequency Offset within the interval ± 0.5, while previously proposed symmetric sequences are limited to ± 0.25. The performance of Frequency Offset and channel estimators has been evaluated by the Cramér–Rao Bound and the theoretical Mean Squared Error. Also, the analytical Mean Squared Error for channel estimation in case of an imperfect estimation of Frequency Offset has been developed to analyze its impact in the performance. Moreover, we have proven that the estimation with the second half of the symmetric preamble performs better than with the whole preamble.

1. Introduction

Multiple-Input Multiple-Output (MIMO)-Orthogonal Frequency Division Multiplexing (OFDM) systems commonly include a sufficiently long Cyclic Prefix (CP) that maintains the orthogonality and thus avoids the appearance of Inter Symbol Interference (ISI) and Inter Carrier Interference (ICI). However, an insufficient CP must be handled in some situations: (i) it can be used to increase the capacity of OFDM systems because its inclusion reduces the systems efficiency, (ii) it can appear in hard wireless scenarios where the channel may be very long, and (iii) certain military applications commonly suppress the CP due to security reasons [1]. In addition, practical MIMO–OFDM systems are highly sensitive to frequency errors because the Frequency Offset (FO) leads to the loss of orthogonality among sub-carriers causing ISI and ICI. Hence, jointly addressing Channel Impulse Response (CIR) and FO estimations in a MIMO–OFDM environment with an insufficient CP has become of major interest.

An extensive literature on MIMO–OFDM channel estimation is already available [2–4]. However, in realistic scenarios, these estimations will be corrupted by an FO which drastically reduces the channel estimator performance [5,6]. In [7–10], a joint channel and FO estimation is proposed for MIMO–OFDM systems. Nevertheless, all these previous algorithms were applied to MIMO–OFDM systems with a sufficiently long CP, so neither ISI nor ICI distorted the signal [11,12].

For Single-Input Single-Output (SISO) systems, some algorithms have been already proposed for channel estimation and iterative cancelation of ISI and ICI in scenarios with insufficient CP [13–17]. For the MIMO case, a CP reconstruction

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algorithm is introduced in [18] and a channel estimator and ISI–ICI cancelation procedure are presented in [19], but their performance decreases in the presence of FO. To our knowledge, the problem of joint channel and frequency offset estimation in MIMO–OFDM systems with insufficient CP has not yet been addressed, and this issue is common in realistic scenarios. Since both are related, they should be addressed together. Hence, in this paper, a joint channel and FO Maximum Likelihood (ML) estimation for MIMO–OFDM systems with insufficient CP is presented. Our proposal makes use of a Pseudo Noise (PN) based preamble with properly designed Symmetric Sequences (SS) for both FO and channel estimation. Moreover, the FO acquisition range has been theoretically derived to demonstrate the limitations of previous training sequences [19]. Additionally, the performance of the estimators has been analyzed by comparison with the Cramér–Rao Bound (CRB) and the Mean Squared Error (MSE); the latter has also been assessed considering the effect of an imperfect FO estimation within the channel estimator. An interesting finding has been that the estimation performs better with the second half of the symmetric preamble than with the whole preamble.

The remainder of this paper is organized as follows. First, in Section 2, the signal model and the joint ML estimator are presented. Section 3 addresses the training sequences and their FO acquisition range is derived. In Section 4, theoretical bounds are derived to validate our results by the CRB and the MSE. In Section 5, simulation results are provided and conclusions are drawn in Section 6.

Notation: Uppercase (lowercase) boldface letters express matrices (vectors), $(\cdot)^H$ represents Hermitian operation, $(\cdot)^*$ denotes the complex conjugate, $|\cdot|$ means absolute value, $\mathbf{0}_{M\times N}$ is an $M\times N$ null matrix, $\mathbf{I}_N$ is an identity matrix of size N, diag$\{\mathbf{x}\}$ is a diagonal matrix with $\mathbf{x}$ as the elements of its main diagonal and zero elsewhere, $\langle\cdot\rangle_N$ stands for the modulo-N operation and Re$(\cdot)$ and Im$(\cdot)$ account for the real and imaginary parts, respectively. We will refer to $\mathbf{x}$ as time-domain vectors and to $\tilde{\mathbf{x}}$ as frequency-domain vectors.

2. Joint ML estimator

A typical $N_t\times N_r$ MIMO–OFDM scheme with N_t transmit and N_r receive antennas will be considered. The frequency-domain symbols are converted to time-domain using an N-length Inverse Discrete Fourier Transform (IDFT) operation, where N is the number of sub-carriers. Assuming an insufficient CP, the received signal from the jth antenna, $j=1,2,\ldots,N_r$, at the ℓth symbol time when the CP is removed can be expressed as

$$\tilde{\mathbf{r}}_j(\ell)=\mathbf{C_o}(\varepsilon)\left[\sum_{i=1}^{N_t}\mathbf{F}_N\mathbf{H}_{ji}\mathbf{F}_N^H\tilde{\mathbf{x}}_i(\ell)+\sum_{i=1}^{N_t}\mathbf{F}_N\mathbf{H}_{ji}^{\mathrm{ISI}}\mathbf{F}_N^H\tilde{\mathbf{x}}_i(\ell-1)-\sum_{i=1}^{N_t}\mathbf{F}_N\mathbf{H}_{ji}^{\mathrm{ICI}}\mathbf{F}_N^H\tilde{\mathbf{x}}_i(\ell)\right]+\tilde{\mathbf{w}}_j(\ell), \tag{1}$$

where $\tilde{\mathbf{r}}_j(\ell)$ is an $N\times 1$ vector with the frequency-domain received signal; $\mathbf{C_o}(\varepsilon)=\mathrm{diag}\{\mathrm{e}^{(1,j2\pi\varepsilon,\ldots,j2\pi\varepsilon(N-1))}\}$ accounts for the frequency offset effect with ε defined as the FO normalized by the sub-carrier spacing Δf; $\mathbf{F}_N$ is the DFT matrix of size $N\times N$; $\mathbf{H}_{ji}$, with $i=1,2,\ldots,N_t$, and $j=1,2,\ldots,N_r$, is an $N\times N$ circulant matrix which consists of the CIRs $\mathbf{h}_{ji}$ of size $L\times 1$ between the ith transmit and the jth receive antennas, where L denotes the channel length and each entry (s,t) is given by $\mathbf{h}_{ij,\langle s-t\rangle_N}$, with $0\le s\le N-1$ and $0\le t\le N-1$; $\tilde{\mathbf{x}}_i(\ell)$ represents an $N\times 1$ vector with the transmitted signal from the ith antenna; $\mathbf{H}_{ji}^{\mathrm{ISI}}$ and $\mathbf{H}_{ji}^{\mathrm{ICI}}$ are the ISI and ICI contributions owing to the previous and current symbols, $\tilde{\mathbf{x}}_i(\ell-1)$ and $\tilde{\mathbf{x}}_i(\ell)$, respectively; and $\tilde{\mathbf{w}}_j(\ell)$ is the $N\times 1$ Gaussian noise component with zero mean and σ_n^2 variance at the jth receive antenna at the ℓth symbol time. First term between brackets in (1) represents the desired signal, while the second and third terms denote the ISI and ICI, respectively. It must be noted that the ICI is not only due to the FO but also to the insufficient CP effect and that the received signal in a realistic system can be influenced by some other effects as the unwanted residual carrier or the DC level inherent in the baseband board chassis. These latter two effects are not considered in this paper but should be taken into account for a more accurate synchronization when this aspect is analyzed.

Let us assume that at the receiver the first half of the preamble, which absorb all the ISI and most part of the ICI, is discarded, then Eq. (1) can be rewritten in time-domain as

$$\mathbf{r}_j(\ell)=\mathbf{C_o}(\varepsilon)\left[\sum_{i=1}^{N_t}\mathbf{H}_{ji}\mathbf{x}_i(\ell)\right]+\mathbf{w}_j(\ell), \tag{2}$$

being $\mathbf{r}_j(\ell)=\mathbf{F}_N^H\tilde{\mathbf{r}}_j(\ell)$ and $\mathbf{x}_i(\ell)=\mathbf{F}_N^H\tilde{\mathbf{x}}_i(\ell)$ two $N\times 1$ vectors with the time-domain received and transmitted signals after the IDFT operation, respectively. Later on this paper the simulation results will demonstrate that the approximation in the above expression is valid. For convenience, the symbol time index ℓ will be omitted in what follows.

Eq. (2) can be rewritten as

$$\mathbf{r}=\mathbf{C}(\varepsilon)\mathbf{X}\mathbf{h}+\mathbf{w} \tag{3}$$

where $\mathbf{r}$ is an $(NN_r)\times 1$ vector whose jth element $\mathbf{r}_j$, with $j=1,2,\ldots,N_r$, is the received signal at the jth receive antenna given by a column N-vector. $\mathbf{C}(\varepsilon)$ is defined as

$$\begin{bmatrix}\mathbf{C_o}(\varepsilon) & \mathbf{0}_{N\times N} & \cdots & \mathbf{0}_{N\times N}\\ \mathbf{0}_{N\times N} & \mathbf{C_o}(\varepsilon) & \ddots & \mathbf{0}_{N\times N}\\ \vdots & \vdots & \ddots & \vdots\\ \mathbf{0}_{N\times N} & \mathbf{0}_{N\times N} & \cdots & \mathbf{C_o}(\varepsilon)\end{bmatrix} \tag{4}$$