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Minimum bit error rate nonlinear precoding for multiuser MIMO and high SNR

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ABSTRACT

This manuscript focuses on the minimization of the bit-error-rate in the high signal to noise ratio regime, for the downlink of a multiuser MIMO channel with N transmit antennas and K single antenna users. In the design of such a precoder the knowledge of the transmitted data and full channel state information at the transmitter are assumed. It is shown that, in the high signal to noise regime, the problem simplifies from a constrained quadratic nonlinear optimization to a single quadratic program, allowing to reduce the complexity. This quadratic problem is equivalent to maximize the minimum distance between the user received symbols and corresponding decision boundaries. The proposed algorithm selects and inverts part of the correlation matrix, unlike the zero-forcing where full inversion is required. This leads to a better performance as the selection allows us to get a better conditioned matrix. Also, this allows us to treat zero-forcing as a special case of the algorithm. The results show that the algorithm achieves a performance close to the optimum, with much lower complexity.

1. Introduction

In recent years, the quest for higher bit rates and the scarcity of spectrum have led to the use of Multiple Input Multiple Output (MIMO) systems as a way out. MIMO in theory can improve the system bit rates linearly with the number of transmit antennas, without resorting to an increase in spectrum usage [1–4]. Even if MIMO systems are well studied for the single user case, that is not the case for the multiuser scenario. For the multiuser scenario, MIMO allows several users to be served with the same space and time/frequency resources. However, to be able to do that some sort of precoding should be used at the transmitter side to separate the user signals, canceling in that way the multiuser interference. The concept of linear multiuser precoding for Single Input Single Output (SISO) systems has been introduced in [5,6]. Such schemes have been proposed to eliminate the multiple access interference and increase the system capacity, while allowing for power allocation strategies. Multiuser linear precoding techniques for MIMO systems have been proposed in [7,8]. A framework based on the convex optimization theory was developed in [9,10] for designing optimum joint linear precoding and post equalization considering full Channel State Information (CSI) [9] and only CSI statistics [10]. The most common and also the lowest complexity schemes available to achieve this type of user separation are Zero-Forcing (ZF) and the Minimum Mean Square Error (MMSE) precoder [11]. Both of them resort to a linear transformation of the data signal, to align it to the corresponding user subspace and to move it away from the subspace of the other users. On the other extreme of complexity we can find Dirty Paper Coding (DPC) [12], which is optimal for the MIMO multiuser Broadcast channel [13,14]. An intermediate solution, in terms of complexity, is the nonlinear minimum Bit-Error-Rate (BER) multiuser transmission scheme (MBMUT). Such a scheme achieves a better BER performance than the linear counterparts at the expense of a higher implementation complexity. The MBMUT scheme was firstly proposed for an SISO Code Division

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Multiple Access (CDMA) system with frequency-selective channels in [15] and was extended to multiple antenna systems in [16]. The fixed power constraint at the transmitter imposes a quadratic constraint into the problem formulation. As the merit function, i.e., the average BER is nonlinear, we are led to a quadratically constrained optimization of a nonlinear function. This can be solved using state-of-the-art nonlinear optimization methods like Sequential Quadratic Programming (SQP), but the complexity is high. To alleviate this in [17] the authors formulated the MBMUT problem by including the power constraint into the merit function, leading to an unconstrained optimization problem. This can be solved using unconstrained optimization techniques [17], but the complexity although somewhat reduced is still high.

In this manuscript, we propose to reduce the complexity of the minimum BER problem by showing that the optimization task can be approximated by a single Quadratic Program (QP). To solve the aforementioned QP a low complexity algorithm is proposed, for generic M-QAM (Quadrature Amplitude Modulation) and we show that the solution has close connections to the linear ZF scheme. Numerical results show that the proposed suboptimal scheme achieve a performance very close to the optimal but with much lower complexity.

This paper is organized as follows. Section 2 describes the system model. In Section 3, a brief revision of the ZF and of the MMSE linear precoders is made. After that, in Section 4 we describe and derive the proposed algorithm to minimize the average system BER and analyze its complexity. In Section 5, the performance of the proposed scheme is evaluated, through numerical simulations. Finally, we conclude the manuscript in Section 6.

Notations: Boldface capital letters denote matrices, boldface lowercase letters denote column vectors. The operation $tr(.)$, $(.)^T$, $(.)^H$ represents the trace, the transpose and the Hermitian transpose of a matrix. $\mathbf{I}_N$, $\mathbf{1}_N$ and $\mathbf{0}_N$ denote an $(N \times N)$ identity matrix, an $(1 \times N)$ all ones column vector and an $(1 \times N)$ all zeros column vector, respectively. $\mathbf{A}_i$ is the ith column of matrix $\mathbf{A}$. $\bar{(.)}$ represents a complex vector or matrix. By $\mathbf{A} \circ \mathbf{B}$ we denote the Hadamard product of matrices $\mathbf{A}$ and $\mathbf{B}$.

2. System model

We consider transmission from a single base station with N transmit antennas to K single-antenna users, as shown in Fig. 1. For such a system the concatenation of all user's received signals, $\bar{\mathbf{y}}(\in \mathbb{C}^{K\times 1})$, can be mathematically described by

$$\bar{\mathbf{y}} = \bar{\mathbf{H}}\bar{\mathbf{x}} + \bar{\mathbf{n}} \tag{1}$$

where $\bar{\mathbf{H}} = [\bar{\mathbf{h}}_1^H, \ldots, \bar{\mathbf{h}}_K^H]^H (\in \mathbb{C}^{K\times N})$ denotes the concatenation of all user channels, $\bar{\mathbf{x}}$ $(\in\mathbb{C}^{N\times 1})$ is the transmitted vector and $\bar{\mathbf{n}}$ $(\in\mathbb{C}^{K\times 1})$ is a vector of independent complex Gaussian noise, with zero mean and variance σ^2. The transmit power at the base station is constrained to unity. The vector of transmitted symbols, $\bar{\mathbf{x}} = [\bar{x}(1), \ldots, \bar{x}(N)]^T$ is obtained by a nonlinear mapping of the vector of M-ary

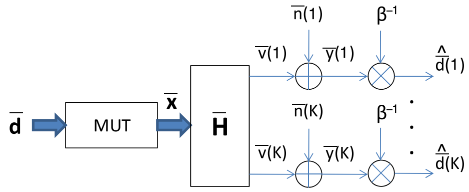

Fig. 1. Block diagram of multiuser transmission.

QAM data symbols $\bar{\mathbf{d}} = [\bar{d}(1), \ldots, \bar{d}(K)]^T$, taken from the odd complex integer grid.

$$\bar{\mathcal{G}} = \{I + jQ|I, Q \in \{\pm 1, \pm 3, \ldots, \pm(\sqrt{M} - 1)\}\}. \tag{2}$$

The real valued representation of a complex vector is obtained by stacking the real and imaginary parts of the corresponding complex vectors:

$$(.) = [\mathcal{R}(.)^T \mathcal{I}(.)^T]^T. \tag{3}$$

With that in mind the system model can be represented by

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n} \tag{4}$$

where $\mathbf{H}$ is the real counterpart of $\bar{\mathbf{H}}$:

$$\mathbf{H} = \begin{bmatrix} \mathcal{R}\{\mathbf{H}\} & -\mathcal{I}\{\mathbf{H}\} \\ \mathcal{I}\{\mathbf{H}\} & \mathcal{R}\{\mathbf{H}\} \end{bmatrix} = \begin{bmatrix} \mathbf{H}_I \\ \mathbf{H}_Q \end{bmatrix} \in \mathbb{R}^{2K\times 2N}. \tag{5}$$

Hereinafter, we will use the real representation of the system, since it is easier to deal with for optimization purposes.

The proposed multiuser precoding scheme assumes perfect CSI at the transmitter side, which can be acquired using the reverse-link estimation in Time-Division Duplex (TDD) or a feedback channel in Frequency-Division Duplex (FDD). Moreover, at the receiver, we assume that before hard decision the received signal is first scaled by a common factor, β^{-1}, at all receivers. This scaling parameter can be easily estimated, by the receivers, without the need to estimate the full CSI. The same is already done, for example, for the linear ZF based scheme. The received signals are scaled, by β^{-1}, so that the average power of the received QAM signals have amplitudes close to the ones of the odd complex integer grid $\bar{\mathcal{G}}$.

3. Linear precoding

In linear precoding the MUltiuser Transmission (MUT) block, as shown in Fig. 1, performs a linear operation, i.e., the input and output symbols are related by a matrix operation ($\bar{\mathbf{x}} = \bar{\mathbf{P}}\bar{\mathbf{d}}$). In this section, we briefly discuss the ZF and MMSE schemes.

3.1. Zero-forcing

This scheme forces that, for each receiver, the contributions from other users are zero, removing all inter-user interference. Mathematically, this corresponds to design the