Topology and its Applications 160 (2013) 2121–2139

Contents lists available at ScienceDirect

Topology and its Applications

www.elsevier.com/locate/topol

Expanding topological space, study and applications

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ARTICLE INFO

Article history:
Received 11 November 2012
Received in revised form 17 August 2013
Accepted 27 August 2013

MSC:
54A05
54A10
54D80
54F65
54H20

Keywords:
Topological space
Fractal manifold
Fractal topology

ABSTRACT

In this paper, we introduce the notion of expanding topological space. We define the topological expansion of a topological space via local multi-homeomorphism over coproduct topology, and we prove that the coproduct family associated to any fractal family of topological spaces is expanding. In particular, we prove that the more a topological space expands, the finer the topology of its indexed states is. Using multi-homeomorphisms over associated coproduct topological spaces, we define a locally expandable topological space and we prove that a locally expandable topological space has a topological expansion. Specifically, we prove that the fractal manifold is locally expandable and has a topological expansion.

1. Introduction

It is known that the space–time of our universe started accelerating its expansion for approximatively 5 billion years [33], where the space–time expansion was confirmed via the galaxies' red shift spectrum [24,10]. The observed recession movement of galaxies was associated to the space–time expansion rather than to the galaxies physical movement (the space expansion creates the recession movement of galaxies, [26,27]). The nature of the space–time expansion (accelerating, decelerating) is generally deduced from observations and modeled by mathematical formalisms. The real nature of the space–time is still unknown and our understanding of its expansion was shaken by the observation of distant supernovae (SNe) of type Ia from a supposed matter dominated space to a vacuum dominated space [34,36,37]. Therefore our theoretical models of the space–time were put into question by observations. Nevertheless the expansion of the space–time is a cosmological reality that still needs to be explored.

One of the major unsolved problems in theoretical physics is the space–time global topology and whether this topology is static or variable together with the space–time expansion. General Relativity is a local

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0166-8641/$ – see front matter
http://dx.doi.org/10.1016/j.topol.2013.08.016

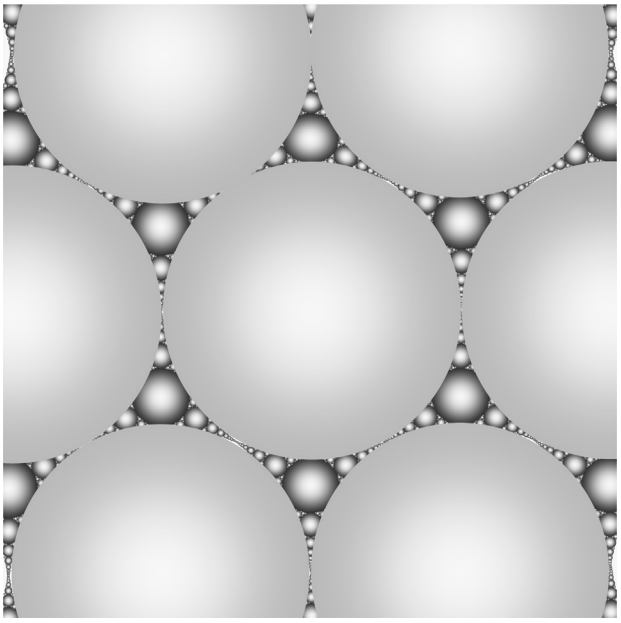

Fig. 1. Illustration of a portion of space–time shape introduced by the fractal manifold model, where the space is defined by an accumulation of packed spheres with variable size [2]. The expansion of the space is generated by the expansion of its basic elements (spheres), and the appearance of expanding Apollonian gaskets between any three packed spheres generates the variation of the space topology.

metrical theory that seems to be incapable to provide information for the global topology of the space–time without topological assumptions [18] (such as asymptotical flatness or asymptotical locally anti-de Sitter [15]). Therefore if the space–time topology is variable, are there any relation between variable topology and space–time expansion or any possible causality? The idea of a change of the space topology was first suggested by J.A. Wheeler [42], and some works were done in this direction [30] based on canonical quantum gravity [8,42]. Others have discussed quantum changes of topology by using the path integral formalism [6,9,17], and different approaches in the literature that relate topology to cosmology can be found in [12,20, 21,28,39,41]. Nevertheless all the previous frameworks in this field are based on metric (including General Relativity, Quantum Gravity, or String Theory) and then the change of topology is only studied locally.

The main objective of this paper is to relate the space expansion to a topological expansion and to provide a new understanding of the space expansion via topology. However relating topology to the continuous deformations of the space, such as its expansion, is quite impossible since topology is invariant under any continuous deformation of the space. This situation calls to circumvent the main difficulty by constructing new topological tools via fractal topology [35] that allow the detection of continuous deformations of space and provide a new formalism for a variable topology without using metrics. The fractal manifold model [1] is taken as an application of these new tools since it is compatible for modeling the space–time expansion by deformations that create holes in a continuous process [2]. Indeed, the space–time described by the fractal manifold is constituted of an infinite number of packed expanding basic elements. The expansion of the space is generated by the expansion of its basic elements, and the appearance of an expanding Apollonian gasket [7,11,23,32] between any three packed basic elements generates the variation of the space topology which is detectable using fractal topology (Fig. 1).

The plan of this paper is as follows: in the preliminary part (Section 2) a general introduction to the fractal topology is presented, followed by an introduction to the fractal manifold and its own fractal topology. Section 3 contains the main result. A definition of topological expansion via local multi-homeomorphisms over coproduct topological spaces is given in Section 3.1.1. We introduce the coproduct topology associated to a fractal family of topological spaces, and we prove that it is expanding in Section 3.1.2. In Section 3.1.3,