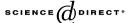


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On the signature of a Lefschetz fibration coming from an involution [☆]

Ki-Heon Yun

Department of Mathematics, Konkuk University, 1 Hwayang-dong, Gwangjin-gu, Seoul 143-701, Republic of Korea

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Abstract

In this article we show that the signature of a Lefschetz fibration coming from a special involution as a product of right-handed Dehn twists depends only on the number of genus on the involution axis. We investigate the geography of such Lefschetz fibrations and we identify it with a blow up of a ruled surface. We also get a geography of the Lefschetz fibration coming from a finite order element of mapping class group as a composition of two special involutions.

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1. Introduction

The study of symplectic topology in dimension four is closely related to the study of Lefschetz fibration over S^2 which is determined by monodromy factorization. A relation in the mapping class group as a product of right-handed Dehn twists gives a monodromy factorization of a Lefschetz fibration.

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Korkmaz [7] found a relation in the mapping class group involving 2g+4 (resp., 2g+10) right-handed Dehn twists when the genus g of the surface is even (resp., odd). Gurtas [5,6] generalized it and he found an involution ϕ of a Riemann surface Σ_g as a product of g+3h+2 right-handed Dehn twists where h is the number of genus on the involution axis. Gurtas asked whether the signature $\sigma(X_{\phi^2})$ of the Lefschetz fibration $X_{\phi^2} \to S^2$ is -4(h+1). It is known that the signature is -4 for h=0 case [7] but it is not known for $h\geqslant 1$.

The article is organized as follows. In Section 2, we briefly review a Lefschetz fibration and a right-handed Dehn twists expression of an involution of type (l, 2k, r) (Definition 4) which was introduced by Gurtas. We show that for a fixed g = l + 2k + r and h = l + r, all involutions of type (l, 2k, r) are related by a sequence of Hurwitz moves and conjugations.

In Section 3, we get a signature formula for the Lefschetz fibration coming from an involution of type (l, 2k, r). Furthermore we also identify such a Lefschetz fibration.

Theorem 1. Let $\phi: \Sigma_g \to \Sigma_g$ be the right-handed Dehn twists expression of an involution of type (l, 2k, r). Let $X_\phi \to D^2$ be the Lefschetz fibration corresponding to ϕ and $X_{\phi^2} \to S^2$ be the Lefschetz fibration corresponding to ϕ^2 . Then $\sigma(X_{\phi^2}) = -4(h+1)$ and $\sigma(X_\phi) = -2(h+1)$. Furthermore, X_{ϕ^2} is diffeomorphic to $(\Sigma_k \times S^2) \# 4(l+r+1) \overline{\mathbb{CP}}^2$.

In Section 4, we study a signature formula of a Lefschetz fibration over S^2 coming from a finite order element of a mapping class group as a composition of two involutions. As an application, we study a p-fold cyclic covering $\pi_p: \Sigma_{p(g-1)+1} \to \Sigma_g$ of the Riemann surface Σ_g and a right-handed Dehn twists expression ψ_p of the $\frac{2\pi}{p}$ rotation map on $\Sigma_{p(g-1)+1}$ which sends the ith handle to the (i+1)st handle of $\Sigma_{p(g-1)+1}$. When p is odd, ψ_p is a composition of two involutions of type (g, (p-1)(g-1), 0). If p is even, then ψ_p is a composition of an involution of type (g, (p-2)(g-1), (g-1)) and an involution of type (1, p(g-1), 0).

Theorem 2. The Lefschetz fibration $X_{\psi_p^p} \to S^2$ is a simply connected 4-manifold which has the following topological invariants:

$$c_1^2(X_{\psi_p^p}) = 4p(p-2)(g-1) \quad and \quad \chi_h(X_{\psi_p^p}) = \frac{1}{8} \big\{ c_1^2(X_{\psi_p^p}) + 4p(g+1) \big\}.$$

2. Preliminaries

Definition 3. Let X be a compact, oriented smooth 4-manifold. A (smooth) Lefschetz fibration is a proper smooth map $\pi: X \to B$ where B is a compact connected oriented surface and $\pi^{-1}(\partial B) = \partial X$ such that

- (1) the set of critical points $C = \{p_1, p_2, ..., p_n\}$ of π is non-empty and lies in int(X) and π is injective on C;
- (2) about each p_i and $\pi(p_i)$, there are local complex coordinate charts agreeing with the orientations of X and B such that π can be expressed as $\pi(z_1, z_2) = z_1^2 + z_2^2$.

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