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Labeled sequent calculus for justification logics

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A B S T R A C T

Justification logics are modal-like logics that provide a framework for reasoning about justifications. This paper introduces labeled sequent calculi for justification logics, as well as for combined modal-justification logics. Using a method due to Sara Negri, we internalize the Kripke-style semantics of justification and modal-justification logics, known as Fitting models, within the syntax of the sequent calculus to produce labeled sequent calculi. We show that all rules of these systems are invertible and the structural rules (weakening and contraction) and the cut rule are admissible. Soundness and completeness are established as well. The analyticity for some of our labeled sequent calculi are shown by proving that they enjoy the subformula, sublabel and subterm properties. We also present an analytic labeled sequent calculus for S4LPN based on Artemov–Fitting models.

1. Introduction

Artemov in [1,2] proposed the first justification logic, called the *Logic of Proofs* (LP), to present a provability interpretation for the modal logic S4 and for the intuitionistic propositional logic. LP extends the language of propositional logic by proof terms and expressions of the form $t : A$, where t is a proof term, with the intended meaning "t is a proof of A." LP can be also viewed as a refinement of the modal epistemic logic S4, in which the knowability operator $\Box A$ (A is known) is replaced by explicit knowledge operators $t : A$ (t is a justification for A). The exact correspondence between LP and S4 is given by the *Correspondence Theorem*: all occurrences of $\Box$ in a theorem of S4 can be replaced by suitable terms to produce a theorem of LP, and vice versa. In light of this theorem, LP is called the justification (or explicit) counterpart of S4. The justification counterparts of other modal logics were developed in [8,10,11,21,24,37,42].

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The combinations of modal and justification logics were introduced in [5–7,18,28]. In this paper, we provide a general definition for combined *modal-justification logics* as follows: a modal-justification logic MLJL is a combination of a modal logic ML and a justification logic JL provided that JL is the justification counterpart of ML. This family of modal-justification logics includes the previous logics introduced by Kuznets and Studer in [28], and in addition contains new combinations.

Various proof systems have been developed for LP (see [2,13,15,20,40,41]), for the intuitionistic logic of proofs [3,38,39], for S4LP and S4LPN (see [14,20,26,41]), and for justification logics of belief (see [23,29]). All of the aforementioned proof systems are cut-free. However, the only proof system claimed to be analytic is Finger's tableaux for LP in [13]. Moreover, most justification logics still lack cut-free proof systems. The aim of this paper will be to present labeled sequent calculi for justification logics, which enjoy the subformula property and cut elimination.

In a labeled sequent calculus some additional information from the semantics of the logic, such as possible worlds and accessibility relation of Kripke models, is internalized into the syntax of the sequent calculus. Thus, in these systems sequents are expressions about the semantics of the logic. We employ Kripke-style models of justification logics, called Fitting models (cf. [4,15,28]), and a method due to Negri [32] to present cut-free G3-style labeled sequent calculi for justification logics. Thus the syntax of our labeled sequent calculi also contains *evidence atoms* for representing evidence functions of Fitting models. Further, we present Fitting models and labeled sequent calculi for modal-justification logics MLJL. In all these labeled sequent calculi, the rules of weakening, contraction, and cut are admissible, and all rules are invertible. Soundness and completeness of the labeled sequent calculi with respect to Fitting models are also proved.

One of the significant properties of proof systems is analyticity. In [19] Gentzen describes the essential property of normal or cut-free proofs as follows: "No concepts enter into the proof other than those contained in its final result, and their use was therefore essential to the achievement of that result." (cf. [19, page 69]). This can be considered as a general description of an analytic proof. The notion of analyticity in the sequent calculus can be formalized by means of the subformula property: every formula in a derivation is a subformula of a formula in the endsequent. However, the subformula property is not necessarily a sufficient condition to guarantee the analyticity of proofs in the labeled systems, since a labeled sequent may contain other ingredients from the semantics of the logic; for example, evidence atoms contain labels, terms, and formulas. The analyticity for some of our labeled sequent calculi is shown by proving that they enjoy the subformula, sublabel and subterm properties. However, in order to obtain full analyticity in the context of justification logics it seems that constant specifications play an important role and they also have to be taken into account. To this end, we relativize the notion of analyticity to constant specifications, and obtain the full analyticity for the labeled sequent calculus of the logic of proofs and its fragments.

The paper is organized as follows. In Section 2, we introduce the axiomatic formulation of modal logics, justification logics, and modal-justification logics. In Section 3, we describe the semantics of justification and modal-justification logics. In Section 4, we present labeled sequent calculi for justification logics, and in Section 5 we establish the basic properties of these systems. In Section 6, we show the analyticity of some of these systems. In Section 7, we prove the admissibility of structural rules, and in Section 8 we prove soundness and completeness of the systems. In Section 9, we present labeled sequent calculi for modal-justification logics. In Section 10, we present a labeled sequent calculus based on Artemov–Fitting models for S4LPN.

2. Modal and justification logics

In this section, we recall the axiomatic formulation of modal and justification logics, and explain the correspondence between them. We also introduce combined modal-justification logics.