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An Analogy Principle in Inductive Logic

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ABSTRACT

We propose an *Analogy Principle* in the context of Unary Inductive Logic and characterize the probability functions which satisfy it. In particular in the case of a language with just two predicates the probability functions satisfying this principle correspond to solutions of Skyrms' 'Wheel of Fortune'.

1. Introduction

Reasoning by analogy is a common feature of inductive inferences, and it has long been seen as desirable that inductive logic capture this kind of argument. The idea behind analogical reasoning is that the probability of seeing a certain kind of object can be positively affected by having seen something that shares some but not all of its properties. For example, we could imagine, as Skyrms [18] does, a wheel of fortune divided into four quadrants: North, South, East and West. If we were to witness the wheel landing on North almost every time, we might want to give greatest probability to it landing on North again, but also a greater probability to it landing on a quadrant adjacent to North (i.e. West or East) than to it landing on the quadrant opposite (South).

Of course in these situations we have a great deal of extra-logical background knowledge, such as the physical mechanics of a wheel of fortune, which surely also influence the (subjective) probabilities we assign. However our intention in this paper is to propose a formalization (our Analogy Principle) of the notion of support by analogy, or nearness, in the absence of any such extra-logical knowledge, and go on to consider

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its consequences. The main result of this paper, Theorem 11, gives a complete characterization of those probability functions satisfying this Analogy Principle (in the presence of certain other standard rationality assumptions) for the case of languages with just two unary predicates, which exactly corresponds to the Wheel of Fortune. We also show in Theorem 13 that for larger (unary) languages only Carnap's probability functions c_0 and c_∞, corresponding to the extreme end points of Carnap's Continuum of Inductive Methods (see for example [1]), satisfy this Analogy Principle.

2. Context and notation

To formalize this idea we work in the conventional (unary) Inductive Logic context. That is, we have a predicate language L_q without equality or functions and having finitely many predicates, $P_1, \dots, P_q$ and constants $a_1, a_2, a_3, \dots,$ the intention being that these enumerate the universe. Let SL_q denote the set of sentences of L_q and $QFSL_q$ the quantifier free sentences of L_q.[2]

In this context, where the a_i are intended to enumerate the universe, we define a *Probability Function on* L_q to be a map $w: SL_q \longmapsto [0,1]$ such that for all $\theta, \phi, \exists x \psi(x) \in SL_q$:

(P1) If $\models \theta$ then $w(\theta) = 1$.
(P2) If $\models \neg(\theta \wedge \phi)$ then $w(\theta \vee \phi) = w(\theta) + w(\phi)$.
(P3) $w(\exists x \psi(x)) = \lim_{m\to\infty} w(\bigvee_{i=1}^m \psi(a_i))$.

Our interest in Inductive Logic is to pick out probability functions on L_q which are arguably *logical* or *rational* in the sense that they could be the choice of a rational agent. Or to put it another way to discard probability functions which could be judged in some sense to be 'irrational'.[3] The usual method of thinning down towards such rational choices is to impose 'rationality principles' which these probability functions should arguably satisfy. Of course there can be considerable disagreement about which principles are rational (to the extent of different candidates being mutually inconsistent, see for example [15]) but one such widely accepted principle is that the inherent symmetry between the constants should be respected by any rational probability function w on L_q. Precisely w should satisfy:

The Constant Exchangeability Principle (Ex). *For $\theta, \theta' \in QFSL_q$, if θ' is obtained from θ by replacing the distinct constant symbols $a_{i_1}, a_{i_2}, \dots, a_{i_m}$ in θ by distinct $a_{k_1}, a_{k_2}, \dots, a_{k_m}$ respectively, then $w(\theta) = w(\theta')$.*

In what follows we take Ex as a standing assumption.

Extending the idea of symmetry between symbols of the language, we might also feel it rational to require that the predicates too are exchangeable.

The Predicate Exchangeability Principle (Px). *For $\theta, \theta' \in QFSL_q$, if θ' is obtained from θ by replacing the distinct predicate symbols $P_{j_1}, P_{j_2}, \dots, P_{j_m}$ in θ by distinct $P_{s_1}, P_{s_2}, \dots, P_{s_m}$ respectively, then $w(\theta) = w(\theta')$.*

A further principle suggested by the idea of symmetry is that of strong negation:

The Strong Negation Principle (SN). *For $\theta, \theta' \in QFSL_q$, if θ' is obtained from θ by replacing each occurrence of $\pm P_i$ by $\mp P_i$ for some predicate P_i, then $w(\theta) = w(\theta')$.*

[2] Some authors, following Carnap (see for example [3]), allow families of mutually exclusive and jointly exhaustive predicates $\{P_1^i, \dots, P_n^i\}$ in the place of the two membered families $\{P_i, \neg P_i\}$. This seems to be motivated by considerations of applications – for example, colours might constitute one such family of properties – in order to slip in extra-logical knowledge about the world and the particular properties the predicates stand for. Since we are interested here in *Pure* Inductive Logic devoid of any such implicit interpretations the languages L_q described seem the most appropriate setting.

[3] It is interesting that 'irrationality' seems much easier to spot than 'rationality'.