Annals of Pure and Applied Logic 165 (2014) 812–836

Contents lists available at ScienceDirect

Annals of Pure and Applied Logic

www.elsevier.com/locate/apal

Observing, reporting, and deciding in networks of sentences

H. Jerome Keisler [a,*], Jeffrey M. Keisler [b]

[a] *Department of Mathematics, University of Wisconsin-Madison, Madison, WI 53706, United States*
[b] *Department of Management Science and Information Systems, University of Massachusetts-Boston, Boston, MA 02125, United States*

ARTICLE INFO

Article history:
Received 11 August 2012
Received in revised form 23 September 2013
Accepted 27 October 2013
Available online 20 November 2013

MSC:
03C40
03F07
68T15
68T27

Keywords:
Craig interpolation
Network
Partition based reasoning
Peer-to-peer inference
Knowledge base

ABSTRACT

In prior work [7] we considered networks of agents who have knowledge bases in first order logic, and report facts to their neighbors that are in their common languages and are provable from their knowledge bases, in order to help a decider verify a single sentence. In report complete networks, the signatures of the agents and the links between agents are rich enough to verify any decider's sentence that can be proved from the combined knowledge base. This paper introduces a more general setting where new observations may be added to knowledge bases and the decider must choose a sentence from a set of alternatives. We consider the question of when it is possible to prepare in advance a finite plan to generate reports within the network. We obtain conditions under which such a plan exists and is guaranteed to produce the right choice under any new observations.

1. Introduction

This paper builds upon the paper [7]. In that paper, a signature network is a network of agents each labeled with a signature. Each agent has a knowledge base within its signature. Unless otherwise stated, all signatures and sentences are understood to be in first order logic. Agents with different but overlapping first order signatures may arise, for example, in an organization where the agents are experts in different areas of specialization, or in distributed networks where a large problem is broken up into small problems to be addressed by agents. A sentence in the language of an agent x is said to be report provable at x if x can verify D after the agents in the network report sentences to their neighbors in their common languages. A signature network is said to be report complete at x if whatever the agent knowledge bases are, every consequence of the union of these knowledge bases that is in the language of x is report provable at x. To

* Corresponding author.
E-mail addresses: keisler@math.wisc.edu (H.J. Keisler), jeff.keisler@umb.edu (J.M. Keisler).
URLs: http://www.math.wisc.edu/~keisler (H.J. Keisler),
http://www.umb.edu/academics/cm/faculty_staff/faculty/jeffrey_m._keisler (J.M. Keisler).

0168-0072/$ – see front matter
http://dx.doi.org/10.1016/j.apal.2013.10.008

obtain conditions for report completeness, the Craig interpolation theorem [4] is applied at each edge in the network.

The present paper considers situations that may arise in applications where there is some systematic relationship among the possible knowledge bases and decision sentences. Specifically, we consider "observation networks" on a given signature network in which there are many potential observations that may be added to the underlying knowledge bases (i.e., learning), and a finite set of alternative sentences one of which must be proved (i.e., making a decision). We exploit the fact that report completeness holds for every knowledge base over a report complete signature network. We do this by introducing the notion of a "report plan" for an observation network — a finite scheme for finding a "report proof" for one of the alternative sentences once observations are added to the underlying knowledge bases. A report plan is decentralized; the agents only need to know their own observations and the reports they receive, not what happens elsewhere in the network. After the observations are made, a report plan is executed by a single pass through the network.

We give a brief preview in Section 2 and a summary of some facts from the literature in Section 3. In Section 4 we discuss single- and multiple-pass report proofs, and in Sections 5 and 6 we introduce the central notions of an observation network and a report plan. Our main result, Theorem 7.3, shows that an observation network will always have a report plan for a given finite set of alternatives provided that: (1) the observation network is sufficient for selecting from the set of alternatives, and (2) the underlying signature network contains a signature tree in the sense of the paper [7]. We also prove two results that are complementary to our main theorem. Theorem 5.6 is a "finiteness theorem" that shows that if an observation network is sufficient for selecting from a given infinite set of alternatives, then some finite part of the observation network is sufficient for selecting from some finite subset of the alternatives. Theorem 7.1 shows that a report plan guarantees that under every possible family of observations by the agents, some alternative has a single-pass report proof. In Proposition 8.5 we briefly indicate a way that report plans might be applied to obtain approximate values for unknown quantities depending on observations. At the end of the paper we give a list of open questions.

2. Preview

Before we formally define the main concepts in this paper, we give in Fig. 1 a simple example that may help the reader to fix ideas. We will refer back to this example later on in this paper. In this example there are five agents arranged in a network as shown in the first picture. At the top is an agent d (the decider) who must decide between the two sentences $A, \neg A$ (indicated by a question mark). Agents y and z may report to w, agent z may also report to x, and agents w and x may report to d. Each agent is equipped with a set of sentences called its knowledge base. In addition, each agent has a set of potential observations that it might make. Before any observations are made, a report plan is prepared as in the picture. This plan gives a set of sentences that might be reported along each edge of the graph. Each agent will make one of its potential observations, independently of the other agents. Then the report plan is executed, building what we call a report proof. This is done with a single pass through the network, beginning at the bottom and ending at the top. Since four agents each have two potential observations, there are 16 possible cases where each agent makes an observation. One such case is shown in the last picture. Beginning at the bottom level, for each edge (r, s) in the graph, agent r proves a sentence from its knowledge base and observation and any sentences reported to it, and reports it to agent s. This sentence must be in the common language of the pair of agents involved. Finally, the decider d at the top is able to prove one of the alternatives, which happens to be $\neg A$ in this case.

We will now formally develop the ideas illustrated in the above example and obtain our results. A central feature is that agents report sentences to other agents in their common language. These sentences are Craig interpolants. Our arguments in this paper will apply results from the paper [7] that in turn depend on the Craig interpolation theorem.