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Algebraic semantics and model completeness for Intuitionistic Public Announcement Logic

Minghui Ma [a], Alessandra Palmigiano [b,*], Mehrnoosh Sadrzadeh [c]

[a] *Institute of Logic and Intelligence, Southwest University, Chongqing, China*
[b] *Faculty of Technology, Policy and Management, Technical University of Delft, The Netherlands*
[c] *School of Electronic Engineering and Computer Science, Queen Mary University of London, UK*

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ABSTRACT

In the present paper, we start studying epistemic updates using the standard toolkit of duality theory. We focus on public announcements, which are the simplest epistemic actions, and hence on Public Announcement Logic (PAL) without the common knowledge operator. As is well known, the epistemic action of publicly announcing a given proposition is semantically represented as a transformation of the model encoding the current epistemic setup of the given agents; the given current model being replaced with its submodel relativized to the announced proposition. We dually characterize the associated submodel-injection map as a certain pseudo-quotient map between the complex algebras respectively associated with the given model and with its relativized submodel. As is well known, these complex algebras are complete atomic BAOs (Boolean algebras with operators). The dual characterization we provide naturally generalizes to much wider classes of algebras, which include, but are not limited to, arbitrary BAOs and arbitrary modal expansions of Heyting algebras (HAOs). Thanks to this construction, the benefits and the wider scope of applications given by a point-free, intuitionistic theory of epistemic updates are made available. As an application of this dual characterization, we axiomatize the intuitionistic analogue of PAL, which we refer to as IPAL, prove soundness and completeness of IPAL w.r.t. both algebraic and relational models, and show that the well known Muddy Children Puzzle can be formalized in IPAL.

1. Introduction

Dynamic logics (DLs) are perhaps the logical formalisms with the widest span of applications. They are designed to describe and reason about *change* brought about by *actions* of diverse nature: updates on the memory state of a computer, displacements of a moving robot in a closed environment, measurements in a model of quantum physics, interactions between cognitive agents performing given communication protocols, belief-revisions changing the common ground between different agents, actions which change the

* Corresponding author.
E-mail addresses: mmh.thu@gmail.com (M. Ma), A.Palmigiano@tudelft.nl (A. Palmigiano), mehrnoosh.sadrzadeh@eecs.qmul.ac.uk (M. Sadrzadeh).

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contextually available referents in a conversation, knowledge update, etc. (the latter ones are examples of *epistemic actions*). In each of these areas, DL-formulas express the properties of the model encoding the given state of affairs, as well as the pre- and post-conditions of a given action. Actions are semantically represented as *transformations* of the current model into another one, which encodes the state of affairs after the given action has taken place. DL-languages are *expansions* of classical (propositional or modal) logic (their *static* underlying logic) with *dynamic operators*, parametrized with actions; the semantic interpretation of a dynamic operator is given in terms of the transformation of models corresponding to its action-parameter.

Similarly to [2], the line of research started with the present paper is motivated by the observations that 'dynamic phenomena' are independent of their underlying static logic being classical, and that this assumption is unrealistic in many important contexts; for instance, in all those contexts (such as scientific experiments, acquisition of legal evidence, verification of programs, etc.) where the notion of truth is *procedural*. In these contexts, affirming ϕ means demonstrating that *some* appropriate *instance* of the procedure applies to ϕ; refuting ϕ means demonstrating that *some* appropriate *instance* of the procedure applies to $\neg\phi$; however, neither instance might be available in some cases. Hence, the law of excluded middle fails in these contexts, and so classical logic is not viable as their underlying reasoning formalism. A more appropriate alternative is e.g. *intuitionistic logic*.

Desirable and conceptually important as it is, the more general problem of identifying the *right* intuitionistic counterparts of (modal-like) expansions of classical logic (such as modal logics themselves, or hybrid logics, etc.) has proven to be difficult, and for most of these logics, this question is still open. Indeed, different axiomatizations which—in the presence of classical tautologies—define the same logic become nonequivalent against an intuitionistic background. Hence, each classical axiomatization might have infinitely many nonequivalent intuitionistic potential counterparts. The most widely accepted proposals of intuitionistic counterparts of given (modal-like) expansions of classical logic have been defined by means of *syntactic* approaches (cf. for instance, the extensive discussion in [22], or more recently [8]), which consist in either weakening the proof systems for the classical versions of these expanded logics so as to make them compatible with the principles of intuitionistic logic, or by defining translations into intuitionistic first-order, or classical propositional languages. However, to the knowledge of the authors, these approaches do not take the *performances* of the given candidate intuitionistic counterpart as the main desideratum, but are rather aimed at establishing *a priori* what the given intuitionistic counterpart should *be*; the performances of the given candidate are then tested, to verify its adequacy.

The main contribution of the present paper is the introduction of a uniform methodology for defining the intuitionistic counterparts of dynamic logics; this methodology is grounded on *semantics* rather than on syntax, and takes *performances* as its main design criterion. As to the first feature, this methodology is based on the dual characterizations of the transformations of models which interpret the actions. For the sake of simplicity, we address one concrete case study, and restrict our attention to the Logic of Public Announcements (PAL), which is one of the simplest yet best known logical framework within the family of Dynamic Epistemic Logics (DELs). PAL was introduced by Plaza in [16] and subsequently intensively studied, both specifically and as part of the DEL-family, viz. [1,10,4] and references therein. As epistemic actions, public announcements correspond to transformations of models which are called *relativizations*. Namely, publicly announcing the formula α corresponds to shifting the given model M to its *submodel* M^α, based on the subset $[\![\alpha]\!]_M$ of the states of M on which α is satisfied.

As mentioned early on, in the present paper, relativization—which is encoded in the injection map $i_\alpha : M^\alpha \hookrightarrow M$—is characterized on algebras via classical Stone duality. Unsurprisingly, this injection map is dually characterized as a certain pseudo-quotient between the complex algebras of the underlying frames of M and of M^α (which, as is well known, are—up to isomorphism—complete atomic BAOs). The advantage brought about by this pseudo-quotient construction is that its definition naturally holds in much more general contexts than the one given by the algebras which are dually equivalent to Kripke frames. These