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The Logic of Uncertain Justifications

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A B S T R A C T

In Artemov's Justification Logic, one can make statements interpreted as "t is evidence for the truth of formula F." We propose a variant of this logic in which one can say "I have degree r of confidence that t is evidence for the truth of formula F." After defining both an axiomatic approach and a semantics for this Logic of Uncertain Justifications, we will prove the usual soundness and completeness theorems.

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1. Introduction

Although Artemov's semantics [1,2] for the Logic of Proofs, his first foray into Justification Logic, make clear its origins in Proof Theory and answering a question left open by Gödel [12], the broad applicability of Justification Logic to nearly all areas in which normal modal logics have application soon became apparent [3,5,6,8,9,18,22–25]. In an epistemic context, Justification Logic takes us from simply $\Box F$, "F is known/believed," to $t : F$, "Reason t provides justification for knowledge/belief that F."

However, not all justifications for belief are equal. One might read about the president's policy speech in the *New York Times* and hear about it from an irate caller on a radio talk show. Each might provide some degree of justification for believing that the president was about to undertake a certain change in policy, but one should put much more credence in the *Times* than in a random caller. We can reflect this by noting the degree p to which a piece of evidence t can serve as justification for the belief that F, which we will notate $t :_p F$.

Note that this does not reflect my degree of belief in F. I might have heard the same thing from both the irate caller on the radio and from the *Times* and the fact that one wouldn't consider the caller a reliable source on this matter doesn't cause one to believe it any less. This distinguishes both our intention and approach from that of logics dealing with the probability that certain propositions are true. See [13] and the many books and papers referenced in its bibliography.

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We will begin with a very short presentation of the basics of Justification Logic, then present the syntax and a Kripke-style semantics for the Logics of Uncertain Justifications, followed by a short discussion of some alternatives to the definitions proposed in the main body of the paper.

2. Justification logic: A brief synopsis

Both to orient the reader and to draw a distinction between what is established and what is new to the present paper, we will begin with a brief overview of the syntax and semantics of (Basic) Justification Logic **J**, the explicit counterpart of the epistemic logic **K** of belief in the absence of introspection. The presentation of definitions and results of this section will be drawn from [4], although the syntax goes back to [2] and the semantics and associated proofs to [10].

The language of **J** is built out of:

- propositional variables P, $Q, \dots$ (possibly with subscripts),
- justification variables x, $y, \dots$ (possibly with subscripts),
- justification constants; we will use c_0, c_1, $c_2, \dots$ as metavariables to stand for justification constants,
- the propositional constant $\bot$,
- the logical connective $\to$,
- the binary functions $\cdot$ and $+$, operating on justification terms, and
- an operator of the type $\langle term \rangle : \langle formula \rangle$, producing formulas.

Justification terms are built up from justification variables and constants by means of the functions $\cdot$ and $+$; *formulas* are built up from propositional variables and $\bot$ using $\to$ (with other connectives defined in the standard way), plus the rule that if t is a justification term and F is a formula, then $t : F$ is a formula.

Intuitively, the formula $t : F$ is to be read as "t represents a justification for believing F." The function $\cdot$ represents the believer's internal application of the justification of belief in the premise of an implication to the justification of belief in the implication itself to justify belief in the consequent of that implication. That is, if s justifies belief in $F \to G$ and t justifies belief in F, then $s \cdot t$ justifies belief in G. The function $+$ combines justifications in the sense that $s + t$ justifies belief in all the things that s justifies belief in as well as in all the things that t justifies belief in.

We will treat constants a little later, but the intuition is that constants justify belief in axiomatically true statements.

Definition 1. The *Basic Logic of Justifications* $\mathbf{J}_0$ consists of the following axiom schemes:

A0. Classical propositional axioms,
A1. $s : (F \to G) \to (t : F \to (s \cdot t) : G)$ (the *application* axiom scheme),
A2. $s : F \to (s + t) : F$ and $t : F \to (s + t) : F$ (the *monotonicity* axiom schemes),

along with the rule of inference *modus ponens*.

Although there can be much subtlety in the treatment of constants ([4,16,20] and many others), they will not be central to our discussion and we will simply extend $\mathbf{J}_0$ to **J** by adding the following rule:

Definition 2. The *Axiom Internalization Rule* states that from any axiom A which is an instance of axiom scheme A0, A1, or A2, for any $n \geqslant 1$ and for any constants $c_1, c_2, \dots, c_n$ we may infer $c_n : c_{n-1} : \dots : c_2 : c_1 : A$.

We will state here only one important syntactic theorem:

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