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Bio mathematical venture for the metallic nanoparticles due to ciliary motion

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ABSTRACT

Background and Objectives: The present investigation is associated with the contemporary study of viscous flow in a vertical tube with ciliary motion.
Methods/Results/Conclusions: The main flow problem has been modeled using cylindrical coordinates; flow equations are simplified to ordinary differential equations using longwave length and low Reynold's number approximation; and exact solutions have been obtained for velocity, pressure gradient and temperature. Results acquired are discussed graphically for better understanding. Streamlines for the velocity profile are plotted to discuss the trapping phenomenon. It is seen that with an increment in the Grashof number, the velocity of the governing fluids starts to decrease significantly.

1. Introduction

In 1962, Sleigh [1] described the tiny microscopic, hair-like structures as Cilia which are found on the surface of mammalian cells. Cilia play a vital part in human and animal development and in everyday life. The length of a single cilium is 1–10 micrometers and its width is even 100 times smaller. Cilia are a source of transportation of fluid. When a group of cilia act together, metachronal waves are produced which carry the fluid from one end to another. These sinusoidal waves produce the effective stroke and the fluid moves in the direction of the wave. Lardner and Shack [2] presented a theory on the transport of fluid through cilia in 1966. Later, Blake [3] presented the model of tiny structures that exist in the ciliated organism. A vast theoretical study on the mechanical properties of transportation of fluid through cilia can be found in Refs. [4–7].

Recently, Jayathilake et al. [8] presented a very nice paper on three dimensional numerical simulations of human pulmonary cilia in the piercingly liquid layer by the immersed boundary method. According to their analysis during the breathing, the airways of the human respiratory system transport air into the lungs. Similarly cilia are present on the majority of mammalian cells throughout development and in adult life of an organism, and have been found to be associated with a variety often overlapping of clinical abnormalities affecting the function of numerous tissues and organs including epithelium of the respiratory tract, oviduct, testes, brain, kidney, eye, inner ear and olfactory epithelium [9]. In fluid mechanics point of view, the study of fluid models of cilia is discussed according to the mucus fluid. Originally, the researchers have considered the Newtonian fluids for the mucus layer while the more realistic or useful mucus layer should be non-Newtonian. There are many non-Newtonian models which exist and not

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Nomenclature

ϵ	Ratio w.r.t cilia length
P	Pressure
r	Variable along the tube
$\overline{\tau}$	Extra stress tensor
λ	Wave length
β_1	Wave number
M	Hartmann number
α	Measure of the eccentricity
a	Radius of the tube
u, w	Velocities
z	Variable normal to the tube
μ	Fluid viscosity
c	Wave speed
β	Heat absorption parameter
B_r	Brickman number
G_r	Grashof number

a single non-Newtonian model satisfies all the properties of the fluids. Therefore, researchers have considered different non-Newtonian fluid models for mucus layer [10,11].

A Nanofluid is a fluid that contains nano-sized particles, called nanoparticles. The nanoparticles used in nanofluids are typically made of metals, oxides, carbides, or carbon nanotubes. Common base fluids include water, ethylene glycol and oil. Nanofluids have many applications in heat transfer, where the nanoparticles are used as a coolant. This attracted many authors to study the heat transfer of nanofluids [12] in a vast set of geometries, such as vertical plates [13], porous stretching sheet [14], and nonlinearly stretching sheet [15]. In modern day mechanics several authors [16–25] have studied the flow behaviors of nanoparticles for different types of nanofluids.

In this paper, we are interested to consider the fluid model for mucus layer. The mechanical properties of the concerned fluid are discussed under long-wavelength and low Reynolds number approximation. In the next section, we describe the formulation of the problem and its simplification to the simplest form. The following sections contain the exact solutions of the formulated problem. In section 4, we study the mechanical properties of the fluid through pictorial.

2. Development of the problem

Here we considered an incompressible *MHD* metallic nanoparticles phenomenon in a symmetric channel. Flow occurs due to the metachronal wave which is produced due to collective beating of the cilia with constant speed c_1 along the walls of the channel whose inner surface is ciliated. The geometry of the problem is presented in the Cartesian coordinate system $(\bar{X},\bar{Y})$ in which $\bar{X}-axis$ lies along the center line of the channel and $\bar{Y}$ is orthogonal to it as shown in Fig. 1:

The geometry of the metachronal wave form proposes that the covering of the cilia advices can be stated precisely as

$$\bar{Y}=f(\bar{X},t)=\pm\left[a+a\epsilon\cos\left(\frac{2\pi}{\lambda}(\bar{X}-c_1t)\right)\right]=\pm L=\pm H. \tag{1}$$

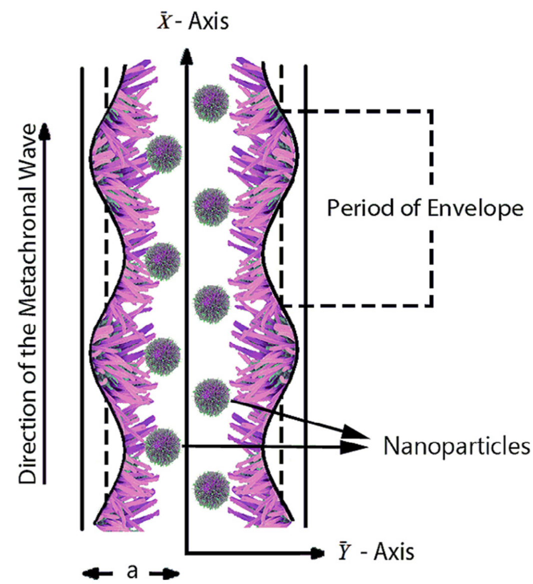

Fig. 1 – Geometry of the problem.

Sleigh et al. [1,6] observed that cilia tips move in elliptical paths; therefore, the vertical position of the cilia tips can be written as

$$\bar{X}=g(\bar{X},t)=X_0+a\epsilon\alpha\sin\left(\frac{2\pi}{\lambda}(\bar{X}-c_1t)\right). \tag{2}$$

The horizontal and vertical velocities of the cilia are given as [1,6]

$$U_0=\frac{-\left(\frac{2\pi}{\lambda}\right)a\epsilon\alpha c_1\cos\left(\frac{2\pi}{\lambda}(\bar{X}-c_1t)\right)}{1-\left(\frac{2\pi}{\lambda}\right)a\epsilon\alpha c_1\cos\left(\frac{2\pi}{\lambda}(\bar{X}-c_1t)\right)}, \tag{3}$$

$$V_0=\frac{-\left(\frac{2\pi}{\lambda}\right)a\epsilon\alpha c_1\sin\left(\frac{2\pi}{\lambda}(\bar{X}-c_1t)\right)}{1-\left(\frac{2\pi}{\lambda}\right)a\epsilon\alpha c_1\sin\left(\frac{2\pi}{\lambda}(\bar{X}-c_1t)\right)}. \tag{4}$$

In the fixed coordinates $(\bar{X},\bar{Y})$ the flow is unsteady. It becomes steady in a wave frame $(\bar{x},\bar{y})$ moving with the same speed as the wave. The transformations between the two frames are:

$$\bar{x}=\bar{X}-c_1t,\ \bar{y}=\bar{Y},\ \bar{u}=\bar{U}-c_1,\ \bar{v}=\bar{V},\ \bar{p}(\bar{x})=\bar{P}(\bar{X},t). \tag{5}$$

With the transformation given in Eq. (5), the equations governing the flow and temperature in the presence of nanoparticle are described as:

$$\frac{\partial\bar{u}}{\partial\bar{x}}+\frac{\partial\bar{v}}{\partial\bar{y}}=0, \tag{6}$$

$$(\bar{u}+c_1)\frac{\partial\bar{u}}{\partial\bar{x}}+\bar{v}\frac{\partial\bar{u}}{\partial\bar{y}}=-\frac{1}{\rho_{nf}}\frac{\partial\bar{p}}{\partial\bar{x}}+\frac{\mu_{nf}}{\rho_{nf}}\frac{\partial^2\bar{u}}{\partial\bar{y}^2}+\frac{\mu_{nf}}{\rho_{nf}}\frac{\partial^2\bar{u}}{\partial\bar{x}^2}-\frac{\sigma B_o^2}{\rho_{nf}}(\bar{u}+c_1), \tag{7}$$

$$(\bar{u}+c_1)\frac{\partial\bar{v}}{\partial\bar{x}}+\bar{v}\frac{\partial\bar{v}}{\partial\bar{y}}=-\frac{1}{\rho_{nf}}\frac{\partial\bar{p}}{\partial\bar{y}}+\frac{\mu_{nf}}{\rho_{nf}}\frac{\partial^2\bar{v}}{\partial\bar{y}^2}+\frac{\mu_{nf}}{\rho_{nf}}\frac{\partial^2\bar{v}}{\partial\bar{x}^2}, \tag{8}$$