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Towards a logical belief function theory

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ABSTRACT

This paper presents a version of belief function theory in which masses are assigned to propositional formulas and which allows the modeler to consider integrity constraints. It also provides three combination rules which apply within this framework. It proves that the initial version of belief function theory and its extension to non-exclusive hypotheses are two particular cases of this proposal. It finally shows that, even if this framework is not more expressive than the belief function theory as defined by Dempster and Shafer, its interest resides in the fact that it offers the modeler a rich language to express its beliefs, i.e., the propositional language.

1. Introduction

Belief function theory is one of the frameworks defined for modeling uncertain information in a quantitative way. As defined by Dempster and Shafer in [5,15], belief function theory considers a frame of discernment which is a finite set of exclusive hypotheses. Some numbers, called masses, are used to represent the extent to which one believes a piece of information and are assigned to elements of the power set of the frame i.e. to unions of hypotheses. Dezert [6] extends this theory by relaxing the assumption of hypotheses exclusivity. Masses are then assigned to elements of a set named the hyper power set of the frame which are intersections of unions of hypotheses.

Cholvy [2] showed that (i) in both cases, the frame of discernment can be seen as a propositional language; (ii) under the assumption of hypotheses exclusivity, expressions on which masses are assigned are equivalent to propositional positive clauses; (iii) when relaxing this assumption, expressions on which masses are assigned are equivalent to some particular kind of conjunctions of positive clauses.

The question we ask in the present paper[1] is the following: why can't we be more general and consider expressions which are equivalent to conjunctions of clauses? I.e., why don't we assign masses to expressions which are equivalent to propositional formulas? This question is motivated by the following example.

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[1] This paper is an extended version of [3].

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Example 1. We consider a witness who reports her observation about a vehicle which passed on the road.

(a) Consider that the witness tells that the vehicle is dark-colored. Suppose that from this testimony alone, I can justify a 0.9 degree of belief that the vehicle is dark-colored.[2] Modeling this leads to consider a frame of discernment with two exclusive hypotheses $\{dark, nondark\}$ and to consider the mass function: $m(\{dark\}) = 0.9$, $m(\{dark, nondark\}) = 0.1$.

(b) Assume now that the witness can distinguish between different types of vehicles. More precisely, she can distinguishes cars, trucks and buses. Suppose that the witness tells she saw a dark-colored car.
Modeling this information within Dempster–Shafer theory can be done by considering 4 exclusive hypotheses: H_1 which represents the fact that a dark-colored car passed, H_2 which represents the fact that a dark-colored truck passed, H_3 which represents the fact that a dark-colored bus passed and H_4 which represents the fact that the vehicle is not dark-colored. My beliefs are then modeled by the mass function: $m(\{H_1\}) = 0.9$, $m(\{H_1, H_2, H_4, H_4\}) = 0.1$.
Modeling this information in the model of [6] leads to consider the frame of discernment $\{dark, nondark, car, truck, bus\}$ in which the pairs of exclusive hypotheses are: $(dark, nondark)$, $(car, truck)$, (car, bus) and $(truck, bus)$ only. My beliefs are then modeled by: $m(\{dark\} \cap \{car\}) = 0.9$, $m(\{dark, nondark, car, truck, bus\}) = 0.1$.

(c) Consider again that the witness can distinguish cars, trucks and buses but assume that she tells that the vehicle she saw is dark-colored but is not a car.
Modeling this information within Dempster–Shafer theory leads to the following mass function: $m(\{H_2, H_3\}) = 0.9$, $m(\{H_1, H_2, H_3, H_4\}) = 0.1$.
Modeling this in the model of [6] leads to consider the mass function: $m(\{dark\} \cap \{truck, bus\}) = 0.9$, $m(\{dark, nondark, car, truck, bus\}) = 0.1$.

Let us come back to point (c). We can notice that in both models the modeler has to reformulate the information she has to model. Indeed, the information provided by the witness is that the vehicle which passed on the road is dark-colored and is not a car. Within the first model, this information is reformulated as: *a dark-colored truck passed or a dark-colored bus passed* (i.e., $\{H_2, H_3\}$). Within the second model, this information is reformulated as: *a dark-colored vehicle, which is a truck or a bus, passed* (i.e., $\{dark\} \cap \{truck, bus\}$). This reformulated expression depends on hypotheses which are not mentioned by the witness. Consequently, in some cases, if the witness happens to distinguish a new hypothesis, this reformulated information must be changed. For instance, if the witness happens to distinguish a new type of vehicles, vans, different form cars, buses and trucks, then the information *a dark-colored vehicle which is not a car passed* must now be modeled by *a dark-colored truck passed or a dark-colored bus passed or a dark-colored van passed* in the first model; and by *a dark-colored vehicle, which is a truck or a bus or a van, passed* in the second. To say it differently, the reformulated information depends also on information which are not the witness' beliefs and which may be not even known by her. Here, it depends on the fact that vans are not cars.

Our suggestion is to offer the modeler a language which allows her to express the very information without reformulation and by using only notions explicitly mentioned by the witness. For doing so, we will abandon set theory for expressing information and use propositional logic instead. More precisely, we will allow the modeler to express information by means of a propositional language. This will lead her to express her beliefs as *any kind of propositional formulas*. In the previous example, this will lead to consider the propositional language whose letters are $dark$, car, $truck$, bus. The information *a dark-colored vehicle which*

[2] Let us mention that, according to Shafer [16], this degree is a consequence of the fact that my subjective probability that the witness is reliable is 0.9. However, in this paper, we do not discuss the meaning of such values nor do we discuss the intuition behind the combination rules.