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Reasoning about evidence

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A R T I C L E I N F O

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A B S T R A C T

Bayesians understand the notion of evidential support in terms of probability raising. Little is known about the logic of the evidential support relation, thus understood. We investigate a number of prima facie plausible candidate logical principles for the evidential support relation and show which of these principles the Bayesian evidential support relation does and which it does not obey. We also consider the question which of these principles hold for a stronger notion of evidential support.

According to standard Bayesian thinking, φ is evidence for ψ iff (if and only if) ψ is more probable conditional on φ than it is unconditionally. Mainstream Bayesians interpret probabilities subjectively, as degrees of belief that are rational in that they obey the probability axioms. Consequently, they understand the notion of evidence as being relativized to particular persons: φ may be evidence for ψ given one person's degrees of belief, but not given another's. In the following, $\rightarrowtriangle_{\mathrm{B}}$ is used to symbolize the Bayesian evidential support relation, and so $\varphi \rightarrowtriangle_{\mathrm{B}} \psi$ means that φ is evidence in the Bayesian sense for ψ; $\varphi \not\rightarrowtriangle_{\mathrm{B}} \psi$ will be used to mean that φ is *not* evidence in this sense for ψ. Pr designates a specific (but unspecified) person's degrees-of-belief function, to which all sentences containing the symbol $\rightarrowtriangle_{\mathrm{B}}$ are taken to implicitly refer. Thus, $\varphi \rightarrowtriangle_{\mathrm{B}} \psi$ is short for $\Pr(\psi \mid \varphi) > \Pr(\psi)$, and $\varphi \not\rightarrowtriangle_{\mathrm{B}} \psi$ is short for $\Pr(\psi \mid \varphi) \not> \Pr(\psi)$.

Relatively little is known about the logical properties of $\rightarrowtriangle_{\mathrm{B}}$. The relation is symmetric: whenever $\varphi \rightarrowtriangle_{\mathrm{B}} \psi$, then also $\psi \rightarrowtriangle_{\mathrm{B}} \varphi$; reflexive for all sentences that are neither fully believed (i.e., believed to a degree of 1) nor fully disbelieved (i.e., believed to a degree of 0): $\varphi \rightarrowtriangle_{\mathrm{B}} \varphi$, provided $\Pr(\varphi) \in (0,1)$; but *not* transitive: the joint truth of $\varphi \rightarrowtriangle_{\mathrm{B}} \psi$ and $\psi \rightarrowtriangle_{\mathrm{B}} \chi$ is compatible with the truth of $\varphi \not\rightarrowtriangle_{\mathrm{B}} \chi$.[1] But suppose $\varphi \rightarrowtriangle_{\mathrm{B}} \psi$ and $(\varphi \wedge \psi) \rightarrowtriangle_{\mathrm{B}} \chi$. Does it follow that $\varphi \rightarrowtriangle_{\mathrm{B}} \chi$? Or, does it hold that $(\varphi \wedge \psi) \rightarrowtriangle_{\mathrm{B}} \chi$ whenever $\varphi \rightarrowtriangle_{\mathrm{B}} (\psi \wedge \chi)$ holds? These and many similar questions seem worth asking, but most of them have no obvious answer. In this paper, I consider a number of principles that are at least prima facie plausible candidate logical principles for $\rightarrowtriangle_{\mathrm{B}}$, and show which of these principles $\rightarrowtriangle_{\mathrm{B}}$ does and which it does not obey.

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[1] Shogenji [22] and Roche [21] study the question of whether the evidential support relation is transitive under certain potentially interesting constraints.

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Some have said that the Bayesian notion of evidence fails to completely capture our intuitive notion of evidence. What we mean when we say that φ is evidence for ψ is—according to these authors—not just that φ makes ψ *more* probable, but also that φ makes ψ *highly* probable. Formally, φ is evidence in this strengthened sense iff (i) $\Pr(\psi \mid \varphi) > \Pr(\psi)$ and (ii) $\Pr(\psi \mid \varphi) > \theta$, for some value θ close, but unequal, to 1. (Different authors hold different views about what the threshold value should be; but all agree—and this will be the only assumption about θ in the following—that $0.5 \leqslant \theta < 1$.) This strengthened evidential support relation—symbolized by $\rightarrowtriangle_{\mathrm{S}}$ in the following—is, like $\rightarrowtriangle_{\mathrm{B}}$, reflexive barring sentences that are either fully believed or fully disbelieved, but it is neither symmetrical nor transitive (Douven [8]). However, that is more or less all that is known about this evidential support relation, as far as its logical properties are concerned. In the following, the same principles that will be considered as candidate logical principles for $\rightarrowtriangle_{\mathrm{B}}$ will also be considered as candidate logical principles for $\rightarrowtriangle_{\mathrm{S}}$.

For candidate principles, I have mined the literature on confirmation theory but also that on conditional logics. The latter might seem an unobvious place to look for such principles, but it is not really—not, at any rate, if there is an intimate connection between conditionals and evidence. For one thing, according to Douven [7] a conditional is acceptable only if its antecedent is evidence for its consequent[2]; but the connection between evidence and conditionals may be even tighter. According to Krzyżanowska, Wenmackers, and Douven [13], not just the *acceptability* but also the *truth* of a conditional requires its antecedent to be evidence (at least in some broad sense) for its consequent.

Be this as it may, *many* of the known principles putatively governing the logic of conditionals make prima facie good sense when interpreted as principles governing the evidential support relation. Many, but not all: for instance, according to the principle commonly known as "Centering," the truth of φ and ψ entails the truth of the conditional "If φ then ψ." Whether or not this is valid as a principle of conditional logic, it makes little sense in our present context: the mere fact that $\varphi \wedge \psi$ holds will in general not imply anything about a person's degrees-of-belief function, and so will not imply anything about whether φ evidentially supports ψ. More generally, it is reasonable to consider only principles whose premises (and also whose conclusion), interpreted in terms of evidential support, constrain a rational person's degrees-of-belief function. Thus we consider only principles whose premises and conclusion are either evidential support statements—statements having $\rightarrowtriangle_X$ as their main operator, with $X \in \{\mathrm{B}, \mathrm{S}\}$—or logical facts (or both).[3]

Table 1 presents the 33 principles to be considered as premise–conclusion rules of the form "Whenever Γ, then φ" (where then for some rules $\Gamma = \emptyset$). Following standard usage, $\supset$ symbolizes the material conditional, $\equiv$ the material biconditional, and $\vdash$ the classical entailment relation. Furthermore, $\perp$ stands for an arbitrary contradiction, and overline notation is used to indicate negation.

It is worth observing that almost all of the candidate adequacy constraints for evidential support relations that were considered as such by Hempel [11] occur in Table 1. Specifically, the principle RCE amounts to Hempel's Entailment Condition for the evidential support relations at issue, RCM is Hempel's Special Consequence Condition, RCEA is his Equivalence Condition, and CNC is his Consistent Selectivity. Hempel also has a Special Consistency Condition, according to which $\varphi \not\rightarrowtriangle \chi$ whenever $\varphi \nvdash \perp$, $\varphi \rightarrowtriangle \psi$, and $\vdash \psi \supset \overline{\chi}$ (here $\rightarrowtriangle$ is used to designate the evidential support relation generically). This is not on the list, and the corresponding principle for the conditional operator has, to my knowledge, never been proposed as an axiom of conditional logic. Note, however, that the Special Consistency Condition is entailed by the conjunction of CNC and RCK.[4] Hempel also had a Converse Consequence Condition, according to which $\varphi \rightarrowtriangle \chi$ whenever $\varphi \rightarrowtriangle \psi$ and $\vdash \chi \supset \psi$. Interpreted as a principle of conditional reasoning, this makes no sense. In fact, it is questionable whether it makes more sense as an adequacy constraint for evidential support, if only because it follows from this condition that any evidence statement that supports some

[2] This has been backed by empirical data in the meantime; see Douven and Verbrugge [9].

[3] Logical facts constrain a rational person's degrees-of-belief function in virtue of the fact that probability respects logic.

[4] From $\vdash \psi \supset \overline{\chi}$ and $\varphi \rightarrowtriangle \psi$ one derives, by RCK, $\varphi \rightarrowtriangle \overline{\chi}$, from which it follows by CNC that $\varphi \not\rightarrowtriangle \chi$.