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On the logical structure of de Finetti's notion of event

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ABSTRACT

This paper sheds new light on the subtle relation between probability and logic by (i) providing a logical development of Bruno de Finetti's conception of events and (ii) suggesting that the subjective nature of de Finetti's interpretation of probability emerges in a clearer form against such a logical background. By making explicit the epistemic structure which underlies what we call *Choice-based probability* we show that whilst all rational degrees of belief must be probabilities, the converse doesn't hold: some probability values don't represent decision-relevant quantifications of uncertainty.

1. Introduction and motivation

This paper tackles the question as to whether the measure-theoretic concept of probability provides a satisfactory quantification of the uncertainty faced by an idealised "rational" agent who is presented with a well-defined choice problem. This is one of the most fundamental questions in the field of uncertain reasoning and as such it has been the focus of heated debates in various disciplines, from the foundations of probability and economic theory, to artificial intelligence. We do not aim at reproducing the many facets of this debate here. However, for the sake of putting our contribution into perspective, we begin by recalling briefly the relevant (to our purposes) interpretations of the concept of "probability" and the decision-theoretic argument supporting its use as a measure of "uncertainty".[1]

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[1] For a compact introduction to the interpretations of probability and the justification of its use in quantifying uncertainty, readers are referred to Williamson [18].

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1.1. Probability and uncertainty

Frequenstist interpretations of probability take the notion of uncertainty as a primitive, and spell it out through the concept of *random-mass phenomena* [15, Ch. 14]. The distinguishing feature of random-mass phenomena is that they are unpredictable in specific details, but predictable in the aggregate. A typical example is the sex ratio of newly born babies. The sex of the next baby to be born at a given hospital is unpredictable, but the country's ratio of males to females tends to be very stable. The focus on random-mass phenomena leads naturally to defining probability as a theoretical limiting frequency. Under this interpretation, probability measures uncertainty as an objective, agent-independent, feature of the world. Subjective interpretations also take uncertainty to be a primitive notion, but refrain from assuming that uncertainty is an objective feature of the world. Hence uncertainty emerges as the psychological state of an agent who is facing a well-defined choice problem, say whether to buy or not an additional travel insurance prior to flying. Under this interpretation, probability is justified as a measure of a rational agent's degrees of belief by making reference to the agent's hypothetical choice behaviour (more on this in the next section).

Measure-theoretic probability, on the other hand, introduces the concept of a probability measure from first principles – Kolmogorov's axioms – which do not refer directly to any underlying interpretation of uncertainty. Standard presentations of the subject (like, e.g. [2,1]) take the probability space Ω to contain all the possible outcomes of some unspecified experiment or observation, but insist that Ω is nothing but a set of points. It is therefore immaterial whether subsets of such points are interpreted as "repetitions" in a random-mass phenomenon (e.g. the sex ratio of newly born babies) or single cases (e.g. getting ill whilst abroad). This neutrality to interpretation may naturally suggest that measure-theoretic probability should be regarded as the unquestionable core of the mathematical representation of uncertainty, for it captures what two otherwise orthogonal interpretations of probability have in common. A consequence of this line of reasoning would then be that any remaining differences are not really of substantial consequence, but merely reflect personal philosophical taste. Whilst a mathematically unified perspective on uncertainty is no doubt appealing, we do believe that the interpretations of uncertainty are of substantial consequence for our formal models. Hence the main purpose of this paper is to show that a logical analysis of the foundations leads to discriminating among formal properties of probability functions *as* measures of uncertainty. Our conclusion will be that not all probability functions serve the purposes of quantifying decision-relevant uncertainty equally well. By articulating this in detail we will put ourselves in the footsteps of the subjective Bayesian tradition, especially Bruno de Finetti's.

From the point of view of de Finetti [7], measure-theoretic probability offers no general justification for applying the *calculus* of probability to reasoning about the uncertainty of single, non-repeatable events. In addition, our ignorance of the boundary conditions of elementary "experiments" or "observations" make the very notion of a "repeatable event" dubious in the least.[2] In reaction to this, de Finetti points out that probability need not arise by making assumptions about the repetition of (independent) events, but is justified by imposing *coherence* to the degrees of belief of an agent who is in a state of uncertainty. Coherence captures all the logico-mathematical properties – essentially, additivity – that a probability function should satisfy to allow for an adequate representation of decision-relevant uncertainty, i.e. *rational degrees of belief.*[3] In short, subjective Bayesianism effectively reduces the meaning of probability to rational choice under uncertainty. This is a central point, which deserves further development.

[2] In the sex ratio example, for instance, it is very difficult to isolate the appropriate reference class for the ratio. The fact that we are observing one hospital gives us only partial information about the population.

[3] In addition, a simple condition like *exchangeability* would suffice to recover the mathematics of probability which naturally springs out of the frequentist definition. This is, in a nutshell, the import of de Finetti's celebrated Representation Theorem – see [14, Ch. 9] for a probability logic version of the result and [16, Ch. 4] for its decision-theoretic interpretation.