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Possibilistic reasoning with partially ordered beliefs

Fayçal Touazi, Claudette Cayrol *, Didier Dubois

IRIT, University of Toulouse, 118 route de Narbonne, 31062 Toulouse, France

A R T I C L E I N F O

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A B S T R A C T

This paper presents the extension of results on reasoning with totally ordered belief bases to the partially ordered case. The idea is to reason from logical bases equipped with a partial order expressing relative certainty and to construct a partially ordered deductive closure. The difficult point lies in the fact that equivalent definitions in the totally ordered case are no longer equivalent in the partially ordered one. At the syntactic level we can either use a language expressing pairs of related formulas and axioms describing the properties of the ordering, or use formulas with partially ordered symbolic weights attached to them in the spirit of possibilistic logic. A possible semantics consists in assuming the partial order on formulas stems from a partial order on interpretations. It requires the capability of inducing a partial order on subsets of a set from a partial order on its elements so as to extend possibility theory functions. Among different possible definitions of induced partial order relations, we select the one generalizing necessity orderings (closely related to epistemic entrenchments). We study such a semantic approach inspired from possibilistic logic, and show its limitations when relying on a unique partial order on interpretations. We propose a more general sound and complete approach to relative certainty, inspired by conditional modal logics, in order to get a partial order on the whole propositional language. Some links between several inference systems, namely conditional logic, modal epistemic logic and non-monotonic preferential inference are established. Possibilistic logic with partially ordered symbolic weights is also revisited and a comparison with the relative certainty approach is made via mutual translations.

1. Introduction

Reasoning with ordered knowledge bases expressing the relative strength of formulas has been extensively studied for more than twenty years in Artificial Intelligence. This concept goes back to Rescher's work on plausible reasoning [38]. But the idea of reasoning from formulas of various strengths is even older, since it goes back to antiquity with the texts of Theophrastus, a disciple of Aristotle, who claimed that the validity of a chain of reasoning is the validity of its weakest link.

* Corresponding author.
E-mail addresses: Faycal.Touazi@irit.fr (F. Touazi), ccayrol@irit.fr (C. Cayrol), dubois@irit.fr (D. Dubois).

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Possibilistic logic [21] is an approach to reason under uncertainty using totally ordered propositional bases. Each formula is assigned a degree, often encoded by a weight belonging to $(0, 1]$, seen as a lower bound on the certainty level of the formula. Such degrees of certainty obey graded versions of the principles that found the notions of belief or knowledge in epistemic logic, namely the conjunction of two formulas is not believed less than the least believed of their conjuncts. This is the basic axiom of degrees of necessity in possibility theory [18].

Deduction in possibilistic logic follows the rule of the weakest link: the strength of an inference chain is that of the least certain formula involved in this chain. The weight of a formula in the deductive closure is the weight of the strongest path leading from the base to the formula. The deductive closure of a base in possibilistic logic corresponds to a total preorder obeying the properties of an epistemic entrenchment [28] on formulas of the classical closure of the base without weights [16]. It is the dual relation to the comparative possibility originally introduced by Lewis [33], and independently retrieved as a counterpart to comparative probability by one of the authors [15], in the context of decision theory.

Possibilistic logic has developed techniques for knowledge representation and reasoning in various areas, such as non-monotonic reasoning, belief revision and belief merging (especially relevant for this paper is [17]); see references in [19,20].

In the last 10 years, only a few authors were interested in extending the possibility theory framework to the partially ordered case, and different approaches have been proposed [30,8,39,4]. In particular, the approach based on partially ordered symbolic weights [4] appears a natural extension of possibilistic logic and looks convenient to implement. It is also worth mentioning the early extension of belief revision theory to partial epistemic entrenchments by Lindström and Rabinowicz [34].

Independently, after the work of Lewis [33] on comparative possibility, conditional logics have been proposed to reason with pairs of formulas linked by a connective expressing relative certainty (or possibility), in a totally ordered setting. Halpern [30] extended relations of comparative possibility to the partially ordered case, studying several ways to extend a partial order on a set to a partial order on its subsets. Moreover, Halpern [30] proposed a conditional logic of partially ordered formulas deriving from a partial order on models.

Following the path opened by Halpern, this paper proposes a simple language, semantics and a proof method for reasoning with partially ordered belief bases, and moreover we compare this approach to possibilistic logic with partially ordered symbolic weights [4]. Before envisaging to reason with partially ordered belief bases, one may wonder where this partial order comes from, and what it means. There are two ways of understanding the lack of completeness of the relation in a partially ordered base:

- *Incomparability*: It reflects the failure to conclude on a preference between two propositions ϕ and ψ, because, according to one point of view, ϕ is preferred to ψ, and from another point of view, the opposite holds. This kind of situation is usual in multiple-criteria decision analysis. We cannot solve this type of incomparability except by modifying the data. See [14] for discussions on the meaning and relevance of the notion of incomparability in decision sciences.
- *Lack of information*: we only know that $\phi > \psi$ is true, but nothing is known for other formulas. In this case the partial order accounts for all total orders that extend it, assuming that only one of them will be correct. This view looks natural if we consider that only partial information about relative strength, for instance of belief, is available, due to lack of time to collect the whole information: the agent expresses only partial knowledge on a subset of propositions he or she finds meaningful.

In the introduction of his paper, Halpern [30] clearly adopts the first view. However, if the relationship $>$ expresses relative certainty, as in our case, one can argue that the second approach is the most natural.

This paper is the follow-up of a previous one [10] that had systematically reviewed the techniques for moving from a partial order on the elements of a set to a partial order on its parts, and systematically