



Computing the Lagrangians of the standard model



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ABSTRACT

Ordinary quantum logic has well known pathologies rendering it useless for the purposes of computation. However, loosely related logics, based upon variants of Girard's Linear Logic, have been found useful in the context of quantum computation. In one sense, the use of such computational schemes affords a meta level view of the possible provenance of certain expressions not otherwise apparent. Since such logics are presumed to encapsulate the essential behavior of quantum "resources" we may entertain the question as to whether this logical or computational approach could have any bearing upon quantum physics itself. In this article we address the question of the genesis of certain fundamental Lagrangians, namely those occurring in the standard model. If a certain set of sentences in a logic are added to the set of axioms of the logic the resulting structure is generally called a *theory* by logicians. In this paper we shall introduce a version of such a logic and deduce some of its physical ramifications. Namely, we will show that there is a single type of sequent that, when added to the logical calculus at hand as an axiom, generates in the theory so defined, series whose leading terms match exactly the Yang–Mills Lagrangian density (including a gauge fixing term) and also the Einstein–Hilbert Lagrangian density, most of the remaining terms being negligible at low intensities in both cases. By expanding the logic somewhat, in the manner of second quantization, we are able also to give an account of interaction terms in the Yang–Mills case. This shows that there is a common form ancestral to all the Lagrangians of the standard model in the ensemble of "evolutionary" trees provided by deductions in a certain clearly specified logic, and reveals the differences between the Yang–Mills and gravitational kinetic terms. Thus we acquire a new paradigm for "unification" of the fundamental forces at the level of the underlying logic.

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1. Introduction

In earlier work [8] the first author attempted to provide a logical basis for quantum computing. The logic used, called **GQ**, was given in the form of a Gentzen sequent calculus which turned out to be identical to a self dual fragment of Girard’s Linear Logic, in its intuitionistic version. (It is shown in [8] that the weak form of quantum logic—namely orthologic, in its intuitionistic form—may be embedded into **GQ**. Orthologic is simply quantum logic without the orthomodular condition.) One advantage of this approach is that **GQ** has a well behaved implication connective, something known to be absent from quantum logic proper. Since this logic is presumed to encapsulate the essential behavior of quantum “resources” we may entertain the question as to whether this logical or computational approach could have any bearing upon quantum physics itself. The present paper is one of a series ([8–10,12]) which addresses aspects of this question.

Specifically, we shall realize certain well-known Lagrangians as outputs of deductions (also called *proofs* in this context) in the theories obtained by adding certain types of simple axioms to the logic at hand.

The layout of the paper is as follows. In Section 2 we briefly discuss our motivation for the adopted axiom, and briefly introduce the logic fragment to be used in this paper. In the following section we review the physical interpretation of deduction in this logic and in Section 4 we put forward our Lagrangian-generating axiom and derive the resulting series in the calculus for the Yang–Mills case. In Section 5 we follow a different deduction made available by the logic in the case of a relativity gauge group to arrive at a series whose leading term is the familiar Einstein–Hilbert Lagrangian (in Palatini form). This choice of a slightly different possible deduction seems to account for the differences between the gravitational Lagrangian and those of the other gauge forces. The derivation of the tracial identities to be used here is relegated to the appended Section 7. In Section 6 we find that the simple setup provided by the logic in the kinetic cases yields a null result in the case of single fermion interactions with gauge fields. This is consistent with the known non-existence of a relativistic quantum theory of fermion–gauge interactions in which particle number is conserved. We introduce the notion of second quantization to extrapolate the algebraic structure deduced in the logic to obtain the appropriate interaction terms as operators consistent with particle non-conservation.

2. The Schwinger Action Principle and a logic of quantum resources

It is our aim in this paper to find some form of precursor to the Lagrangians of the standard model in a sense to be made precise below. To gain an idea of the general forms possible, we recall the Schwinger Action Principle for a given Lagrangian density $\mathcal{L}(\varphi^I)$, where φ^I denotes a tuple of fields defined on spacetime, and in the expression below, σ_0, σ_t denote two space-like surfaces time-like separated in the order specified, and $|\varphi^I(\sigma_0)\rangle, |\varphi^I(\sigma_t)\rangle$, denote the states representing the values of the tuple φ^I on the respective surfaces, suitably normalized. Then, in units chosen so that $\hbar = c = 1$, the Schwinger Action Principle reads:

$$\delta \langle \varphi^I(\sigma_t) | \varphi^I(\sigma_0) \rangle = i \langle \varphi^I(\sigma_t) | \delta \left(\int_P \mathcal{L} d^4x \right) | \varphi^I(\sigma_0) \rangle. \quad (1)$$

That is to say, the variation of the amplitude when the field configurations along a path P are varied is i times the value obtained by sandwiching the concomitant variation of the action integral along P , between the initial and final states.

If we carry out the variation over the initial configuration only, this yields

$$\langle \varphi^I(\sigma_t) | \delta | \varphi^I(\sigma_0) \rangle = i \langle \varphi^I(\sigma_t) | \delta \left(\int_P \mathcal{L} d^4x \right) | \varphi^I(\sigma_0) \rangle \quad (2)$$

so that we surmise, upon the formal cancellation of δ with \int_P , that

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