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Multiple-Valued Logic mathematical approaches for multi-state system reliability analysis

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A R T I C L E I N F O

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A B S T R A C T

A mathematical description of an examined system such as Multi-State System (MSS) permits the system reliability to be analyzed in more detail, because the MSS defines some performance levels (more than only working and failure). The structure function is one of the basic definitions and representations of MSS. But the dimension of the structure function increases critically depending on the number of system components. Therefore, the development of methods for examination and quantification of such a function is an actual problem in MSS reliability analysis. In this paper, a method for the analysis of the MSS structure function of high dimension is proposed. The principal point of this method is the interpretation of the MSS structure function as a Multiple-Valued Logic function. It allows effective and approved mathematical methods of Multiple-Valued Logic to be used for analysis and quantification of the MSS structure function. We propose to use two mathematical approaches of Multiple-Valued Logic for the MSS. One of them is a representation of the MSS structure function by a Multiple-Valued Decision Diagram. It is an effective approach for analysis and estimation of the function of high dimension in Multiple-Valued Logic. The other approach is Logic Differential Calculus. Logic Differential Calculus is a useful approach for analysis of the MSS state changes.

1. Introduction

A mathematical representation and description of the initial object is an important step in reliability analysis. There are two principal mathematical models in reliability analysis as shown in [2,23]. The *Binary-State System* (BSS) allows for the investigating of two states for the initial system: functioning and failure. The *Multi-State System* (MSS) is a mathematical model that is used to describe a system with several (more than two) levels of performance. A detailed analysis of the methods of MSS reliability estimation and quantification is presented in [14,23]. The authors of these papers show a number of examples of MSS application in reliability analysis of information, manufacturing, production, power generation, transportation, and other systems. They have considered algorithms for the MSS estimation and calculation of many indices of MSS reliability and availability.

The MSS importance analysis is one of the possible directions for the estimation of MSS behavior against the system structure and components states. This analysis allows the examining and quantifying the influence of change of the component state on the system performance level. Therefore, every system component has a measure of its importance for the system functioning and failure. This measure in reliability engineering is called *Importance Measure* (IM). IM is a probability of the MSS performance level change depending on the change of the system component state [9,14]. There are different methods and algorithms for the calculation of IMs for the MSS. In paper [8] IMs, a system with two performance levels and

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multi-state components is considered. J.E. Ramirez-Marquez and D.W. Coit in [16] have generalized this result for MSS and have proposed a new type of IM called the composite importance measures. An actual review of IMs is presented in [10]. A new application of IMs for degraded MSS has been proposed in [15]. New methods for importance analysis of MSS are considered in [19,21]. These methods are based on the mathematical approach of *Multiple-Valued Logic* (MVL) such as Logical Differential Calculus. The Logical Differential Calculus is a mathematical approach that permits changes in MVL function to be analyzed depending on changes of its variables [17]. Therefore, this tool can be used to evaluate the influence of every system component state change on the MSS performance level if the MSS structure function is interpreted as a MVL function [21]. But the Logical Differential Calculus computational complexity increases depending on the number of system components. The computational complexity of the MVL function analysis is a typical problem with MVL. Therefore, there are different investigations for a solution to this problem. Multiple-Valued Decision Diagrams (MDDs) is one of the effective methods for the representation of the MVL function of large dimensions [12].

MDD is a generalization of a Binary Decision Diagram (BDD). A BDD is widely and efficiently used in reliability analysis for BSS [4,5]. There are some investigations of BDD application in reliability analysis of the MSS [3]. In this case every performance level of the MSS is considered and analyzed independently that causes the additional transformation of the MSS, and the integral analysis of the system is insufficient. Therefore, the MDD is a natural extension of BDD for MSS analysis [22,18]. In [1] the MDD has multi-valued/multi-state nodes and is constructed for every performance level. This MDD is formed by the transformation of a Fault Tree to the MDD. The MDD proposed in [22] represents and defines all the system performance levels. The MSS representation by MDD causes the development of new algorithms for system reliability analysis. Algorithms for the frequency indices calculation (Mean Time Between Failure, Mean Time To Failure, Mean Time To Repair) based on MDD are proposed in [1,18]. The calculation of some IMs by the MDD is considered in [6]. In this paper we develop the result of [20,22] and propose a new MDD-based method for the MSS analysis using Logical Differential Calculus for the MSS importance analysis. This method is based on the transformation of the MSS structure function to an MDD that permits the initial object to be represented definitely.

Section 2 introduces concepts of MSS structure function, Logical Differential Calculus and MDD. Section 3 illustrates the calculation of IMs by Logical Differential Calculus. In this section the IMs definitions in terms of Logical Differential Calculus are considered. The hand-calculation example illustrates the clarity of using Logical Differential Calculus for MSS importance analysis. Section 4 presents algorithms for the MSS quantification based on MDD and considered IM. These algorithms are founded on two mathematical tools of MVL: Logical Differential Calculus and MDD. A benchmark study is presented to illustrate the proposed method.

2. Preliminaries

2.1. Representation of MSS

Consider the MSS of n component that has m performance levels [2,7]. The MSS performance level changes from zero to $(m-1)$. Each of the n system components can be in one of m possible states: from complete failure (denoted by 0) to perfect functioning (denoted by $m-1$). A *structure function* is one of the typical representations of the MSS and defines the correlation of the system performance level depending on MSS components states [7,9,19]:

$$\phi(\boldsymbol{x})\colon \{0,\ldots,m-1\}^n \rightarrow \{0,\ldots,m-1\}, \tag{1}$$

where x_i is the i-th component state; $\boldsymbol{x}=(x_1,\ldots,x_n)$ is the vector of components states.

Every system component state x_i is characterized by the probability of the performance rate:

$$p_{i,s}=\Pr\{x_i=s\}, \quad s=0,\ldots,m-1. \tag{2}$$

The MSS structure function (1) allows some types of description. As a rule three types of description are used (Fig. 1): (a) analytical, (b) tabular, and (c) graphical. The analytical description implies definition of the MSS structure function by a mathematical formula (logical or arithmetical). The tabular description is a definition of the correlation of the function value and values of function variables values by table called the Truth Table. The MDD is used as the graphical definition of the structure function and is a directed acyclic graph.

There are other representations of MSS besides the structure function (1). For example, the *Multi-state Fault Tree* (MFT) and the *Multi-state Reliability Block Diagram* (MRBD) are considered as typical representations of the MSS in [1]. These forms of system representation are widely used in reliability analysis of BSS.

The MFT is a mathematical and graphical interpretation of combinations of events that can cause the MSS to occupy a specific state [1]. Therefore, the MFT must be built for every system performance level (Fig. 2). The top event of each MFT is related to the system being in a specific state s $(s=0,\ldots,m-1)$. The top event is obtained by the combination of basic events that can cause the occurrence of s by means of AND, OR, and k-out-of-n logic gates. Each basic event in the MFT is a specific component state. The quantitative analysis of MFT is used to determine the probability of the system being in that specific state depending on the occurrence probabilities of basic events.

An MRBD is another technique for the MSS reliability estimation. This technique is widely used in reliability analysis of a BSS and is a success-oriented network describing the functions of a system. An MRBD consists of input point, an output