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A cirquent calculus system with clustering and ranking ☆

Wenyan Xu

School of Mathematics and Statistics, Xidian University, Xi'an 710071, China

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ABSTRACT

Cirquent calculus is a new proof-theoretic and semantic approach introduced by G. Japaridze for the needs of his theory of computability logic (CoL). The earlier article "From formulas to cirquents in computability logic" by Japaridze generalized formulas in CoL to circuit-style structures termed cirquents. It showed that, through cirquents with what are termed clustering and ranking, one can capture, refine and generalize independence-friendly (IF) logic. Specifically, the approach allows us to account for independence from propositional connectives in the same spirit as IF logic accounts for independence from quantifiers. Japaridze's treatment of IF logic, however, was purely semantical, and no deductive system was proposed. The present paper syntactically constructs a cirquent calculus system with clustering and ranking, sound and complete w.r.t. the propositional fragment of cirquent-based semantics. Such a system captures the propositional version of what is called extended IF logic, thus being an axiomatization of a nontrivial fragment of that logic.

1. Introduction

Cirquent calculus is a new proof-theoretic and semantic approach introduced by G. Japaridze [5] for the needs of his theory of computability logic [4,7]. Its main characteristic feature is being based on circuit-style structures called *cirquents*, as opposed to the more traditional approaches that manipulate tree-like objects such as formulas. Cirquents, unlike formulas, allow (one or another sort of) *sharing* of subcomponents between different components. Due to sharing, cirquent calculus has greater expressiveness and higher efficiency. For instance, as shown in [6], the analytic cirquent calculus system CL8 achieves an exponential speedup of proofs over the traditional analytic systems. Since its birth, cirquent calculus has been developed in a series of articles [1,6,8–10,15–18].

The concept of cirquents was qualitatively generalized in [8], where the ideas of *clustering* and *ranking* were introduced. Intuitively, clusters are generalized disjunctive or conjunctive gates, namely, switch-style

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E-mail address: xwykiki@126.com.

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devices that combine tuples of individual gates of a given type in a parallel way—in a way where the choice (*left* or *right*) of an argument is shared between all members. Ranks are seen to be consoles of certain subsets of clusters of a given type, with all such consoles arranged in a linear order indicating in what order selections by the consoles should be made. It was showed semantically in [8] that, through cirquents with clustering and ranking, one can capture, refine and generalize *independence-friendly (IF) logic* [3,11]. The latter, introduced by J. Hintikka and G. Sandu, is a conservative extension of classical first-order logic, whose main distinguishing feature is allowing one to express independence relations between quantifiers. The past attempts (cf. [12,13]) to develop IF logic at the purely propositional level, however, have remained limited only to some special syntactic fragments of the language.

Japaridze's treatment of IF logic in [8] was purely semantical, and no deductive system was proposed. In this paper, we axiomatically construct a cirquent calculus system called RIF_p, with clustered disjunctive and conjunctive connectives and ranks, sound and complete w.r.t. the propositional fragment of Japaridze's cirquent-based semantics. Such a system captures the propositional version of what is called *extended IF logic* (cf. [14]), thus being an axiomatization of a nontrivial fragment of that logic. The earlier article [18] constructed a cirquent calculus system called IF_p which, with only clustered disjunctive connectives, is an axiomatization of propositional, non-extended IF logic. RIF_p is a conservative extension of IF_p, namely, the latter is the fragment of the former limited to cirquents with ≤ 2 ranks where all $\vee$-clusters are in the lowest rank, all $\wedge$-clusters are in the highest rank and all $\wedge$-clusters are singletons. Thus the present paper substantially strengthens the results of [18].

2. Preliminaries

In this section we reproduce the basic concepts from [8] on which the later parts of the paper will rely. An interested reader may consult [8] for additional explanations, illustrations and examples.

Our propositional language has infinitely many **atoms**, for which $p, q, r, s, \ldots$ will be used as metavariables. An atom p and its negation $\neg p$ are called **literals**. A **formula** means one of the language of classical propositional logic, built from literals and the binary connectives $\wedge$, $\vee$ in the standard way. $A \to B$ is understood as an abbreviation of $\neg A \vee B$. And $\neg$, when applied to anything other than an atom, is understood as an abbreviation defined by $\neg\neg A = A$, $\neg(A \wedge B) = \neg A \vee \neg B$ and $\neg(A \vee B) = \neg A \wedge \neg B$. Namely, all formulas are required to be in negation normal form.

A **cirquent**[1] is a formula together with two extra parameters called **clustering** and **ranking**, respectively, where:

- Clustering is a partition of the set of all occurrences of $\vee$, $\wedge$ into subsets, called **clusters**, satisfying the condition that all occurrences of $\vee$, $\wedge$ within any given cluster have the same type (all are disjunctive, or all are conjunctive). A cluster containing disjunctive connectives is said to be a **$\vee$-cluster**, and a cluster containing conjunctive connectives a **$\wedge$-cluster**. With each cluster is associated a unique positive integer called its **ID**. IDs serve as identifiers for clusters, and we will simply say "cluster k" to mean "the cluster whose ID is k".
- Ranking is a partition of the set of all clusters into subsets, called **ranks**, arranged in a linear order, with each rank satisfying the condition that all clusters in it have the same type (all are $\vee$-clusters, or all are $\wedge$-clusters). A rank containing $\vee$-clusters is said to be **disjunctive**, and a rank containing $\wedge$-clusters **conjunctive**. With each rank is also associated a unique positive integer called its **ID**, satisfying the

[1] The concept of cirquents considered in cirquent calculus is more general than the one defined here. See [8].