Future determination of entities in Talmudic public announcement logic ☆

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A R T I C L E   I N F O

A B S T R A C T

Ordinary dynamic action logics deal with states and actions upon states. The actions can be deterministic or non-deterministic, but it is always assumed that the possible results of the actions are clear cut.

Talmudic logic deals with actions (usually legally meaningful actions which can change the legal status of an entity) which depend on the future and therefore may be not clear cut at the present and need future clarifications.

The clarification is modelled by public announcement which comes at a later time after the action has taken place.

The model is further complicated by the need to know what is the status of formulas at a time before the results of the action is clarified, as we do not know at which state we are in. Talmudic logic treats such states much like the quantum superposition of states and when clarification is available we get a collapse onto a pure state.

The Talmudic lack of clarity of actions arises from applying an action to entities defined using the future, like the statement of a dying man on his death bed:

> **Let the man who will win the jackpot in the lottery next week be the sole heir in my will now**

We need to wait a week for the situation to clarify.

There is also the problem of legal backwards causality, as this man, if indeed he exists, unaware of his possible good fortune, may have himself meanwhile donated all his property to a charity. Does his donation include this unknown inheritance?

This paper will offer a model and a logic which can represent faithfully the Talmudic reasoning in these matters.

We shall also see that we get new types of public announcement logics and (quantum-like) action logics. Ordinary public announcement logic deletes possible worlds after an announcements. Talmudic public announcement logic deletes accessibility links after an announcement. Technically these two approaches are similar but not equivalent.

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1. Introduction and orientation

This paper offers a new variant of public announcement logic arising from the logical modelling of the Talmudic approach to the legal status of entities defined by future conditions.

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3 possible presidents

6 November 2012
US Presidential elections

27 October 2011

Warren Buffet and Bill Gates give a gift today
of total 50 billion US dollars to the candidate
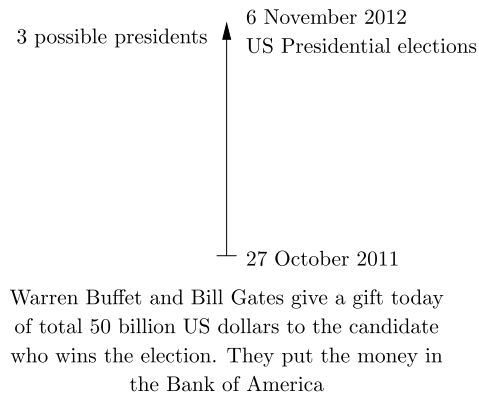who wins the election. They put the money in
the Bank of America

Fig. 1. Donation.

We present and discuss the Talmudic legal options in Example 1. We ask the reader to accept that our analysis and presentation of these Talmudic options is correct (see our monograph [1] on the subject) and focus our analysis on what kind of logic is required to formalize these options.

Readers more interested in logic as applied to law are invited to read up to Section 3 of this paper and further consult [1] and [2].

Readers interested in formal logic and in traditional public announcement logic are invited to continue to read Sections 4 and 5 of our paper and to consider our system as a new variant (hopefully of some merit) of traditional public announcement logic. We note that such a logic has applications not only in formalizing Talmudic reasoning but also to general modern legal reasoning as well. Our strategy is as follows:

(A) First we give the reader a general impression of what kind of a new logic to expect.
(B) Then we give a motivating example allowing us to present and discuss the Talmudic approaches requiring a new logic.
(C) The remainder of Section 1 is devoted to a semi-formal discussion of the logical components required.
(D) Sections 2–3 describe the new logic in a semi-formal manner. Section 4 is more aimed at the formal logician and formally describes Propositional Talmudic public announcement logic TPK.
(E) Section 5 compares the new logic with traditional public announcement logic.
(F) Appendices A and B collect various technical results, to which we can refer in the appropriate places in the paper.

Our starting point is the Kripke semantics for predicate modal logic **K** with expanding domains. This semantics has models of the form $\mathbf{m} = (S, R, \mathbf{m}_t)$, t in S, where S is the set of possible worlds, R is a binary relation on S and for each t in S, $\mathbf{m}_t$ is a classical model. We require that whenever tRs holds then the domain D_t of $\mathbf{m}_t$ is a subset of the domain D_s of $\mathbf{m}_s$.

The traditional way of adding a public announcement feature to such models is to allow for operators of the form $[\phi]$, where ϕ is a formula of the language, which takes a model $\mathbf{m}$ to a model $\mathbf{m}_\phi$, whose set of possible worlds is $S_\phi = \{t \in S \mid t \vDash \phi\}$.

Let us isolate the two features we are going to change in this semantics

(a) The kind of classical models $\mathbf{m}_t$ we are going to use is going to be special and the embedding of the domain of $\mathbf{m}_t$ in the domain of $\mathbf{m}_s$ (for the case of tRs holding) is also special. This restricts the class of models of the semantics.
(b) While traditional public announcement operators take out points elements of S, the Talmudic public announcement operators take out elements of R. These two operations are not exactly equivalent (see Appendix B.4).
(c) Note that the changes in (a) and in (b) above are orthogonal and independent of each other. So once we explain the change (a) for the classical models (which we call Talmudic classical models), we can explain the new Talmudic public announcement logic of change (b) on the propositional level only.

We now motivate the need for above changes via an example.

Example 1 *(Presidential elections).* Warren Buffet and Bill Gates give a gift on 27 October 2011 of a total of 50 billion US dollars to the candidate who wins the presidential elections (see Fig. 1). They put the money in an account in the Bank of America. We assume for simplicity that there are at this date 3 candidates running and so we can assume that by 6 November 2012 we know who wins. The question arises what is to be done with this donation in the period between 27 October 2011 and 6 November 2012 (see Fig. 2).

The legal question we ask, under the above circumstances, who has control of the money in the periods between 27 October 2011 and 6 November 2012 and in the period after 6 November 2012?