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The categorical imperative: Category theory as a foundation for deontic logic

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ABSTRACT

This article introduces a deontic logic which aims to model the Canadian legal discourse. Category theory is assumed as a foundational framework for logic. A deontic deductive system $\mathcal{DDS}$ is defined as two fibrations: the logic for unconditional obligations $\mathcal{OL}$ is defined within a Cartesian closed category on the grounds of an intuitionistic propositional action logic $\mathcal{PAL}$ and an action logic $\mathcal{AL}$, while a logic for conditional normative reasoning $\mathcal{CNR}$ is defined as a symmetric closed monoidal category. A typed syntax and typed arrows are used to define properly $\mathcal{DDS}$. We show how it can solve the paradoxes of deontic logic and we provide some examples of application to legal reasoning.

1. Introduction

Deontic logic was introduced in analogy with modal logic by von Wright [119] to model normative reasoning.[2] After the developments of possible world semantics with the work of Hintikka, Montague and Kripke (see [126]), von Wright's initial approach was redefined within the framework of modal logic. This gave rise to the well-known standard system of deontic logic, the modal logic *KD*. Many objections were raised against von Wright's initial approach,[3] but Chisholm's [38] paradox was the most damaging to the standard systems. It showed that monadic deontic logics cannot properly model conditional normative reasoning, which is central to the normative discourse.

Chisholm's objection was followed by various proposals. Some authors argued that contrary-to-duty reasoning should be modeled through a dyadic framework, specifying the conditions under which the obligations hold (see for example van Fraassen [118], Al-Hibri [2]).[4] Others argued that Chisholm's paradox arises because standard systems do not take into account the temporal dimension implicit to conditional

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[2] See [46,83] for an introduction.

[3] See [83] for an overview.

[4] Historically, the building blocks of dyadic deontic logic were introduced by [120] in answer to Prior's [98] paradox of derived obligation. Dyadic deontic logic has however been used as a solution to Chisholm's puzzle.

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normative reasoning (e.g., [43]), and this led to the introduction of temporal deontic logics (see for example Thomason [111], van Eck [116,117]). But still, other issues remained with these approaches, such as their inability to properly model conflicting obligations and factual detachment. This led some authors to introduce different solutions to answer the problems of detachment of deontic conditionals and, more importantly, to model conflicting conditional obligations. In addition to Makinson and van der Torre [80–82], who introduced input/output logics to model normative conditional and unconditional reasoning, one can find various proposals in non-monotonic (see [86] for an introduction) and adaptive logics (see for example [109,15,14]).

Even though deontic logic was first meant as a formal framework to analyze and evaluate normative reasoning, it has since then been used to serve different purposes. In addition to the analysis of inferences, deontic logic has been used to model normative systems (e.g., [76,35,21]) and multi-agent normative systems (e.g., [102,103,61,88,57,26]). It has been used to model contracts (e.g., [20,28,100]) and obligations with deadlines (e.g., [45,27,44]). It has been used in computer science (e.g., [125,114,34,33,19]), artificial intelligence (e.g., [24,23]) and in law (e.g., [104,124,97,54,18]).

This list does not pretend to be exhaustive and is in all likelihood incomplete. There is, however, a lesson that should be learned from this diversity: a system of deontic logic cannot be criticized independently of its purpose. For instance, a deontic logic which aims to model the evolution of a computer program will not require the same characteristics as one that tries to model the structure of the law. Similarly, a deontic logic that aims to model contracts will not need the same properties as one that models inferences. In the present paper, our aim is to introduce a deontic logic adequate to analyze and evaluate the structure of legal inferences. The aim is not to develop a deontic logic that can represent the structure of the Canadian legislation, nor to develop a formal system that can model how legal reasoning is done. Rather, our objective is to develop a formal system that can help in the analysis of the structure of legal reasoning, specifying how it should be done.

The results of the present paper are built upon previous work. Following the seminal work of Lambek [70] and what was presented in [95], we introduce a typed deontic logic within the framework of categorical logic. There are three main theoretical motivation for this paper. First, our aim is to introduce an alternative framework to modal logic to model unconditional obligations. Secondly, we wish to introduce an alternative framework to Boolean algebras to model actions within deontic contexts. Finally, the objective is to introduce a monadic formal system able to model conditional normative reasoning and conflicting obligations without requiring the techniques of non-monotonic or adaptive logics. As such, the deontic logic we propose will operate at three different levels. In a nutshell, we propose to define a deontic deductive system $\mathcal{DDS}$ on the grounds of an action logic $\mathcal{AL}$ and a propositional action logic $\mathcal{PAL}$ (cf. [90]), an obligation logic $\mathcal{OL}$ (cf. [94]) to model unconditional obligations and a logic $\mathcal{CNR}$ that can model conditional normative reasoning with a monadic O (cf. [93]).

The structure of the paper is as follows. In the next section, we present the rationale of our framework and expose the characteristics that a deontic logic which aims to model the Canadian legal discourse should satisfy. This will be followed by a brief exposition of the foundational framework we adopt. Then, in Section 3, we present all the relevant material that is required to define $\mathcal{DDS}$, and we provide the categorical definition in Section 4. The semantics is presented in Section 5, and a comparison of $\mathcal{DDS}$ with Goble's [51] analysis is provided in Section 6. Then, a discussion of some paradoxes is provided in Section 7, where we provide examples of applications of $\mathcal{DDS}$ to the Canadian legal discourse. We conclude in Section 8 with remarks for future research.