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A logic for diffusion in social networks

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ABSTRACT

This paper introduces a general logical framework for reasoning about diffusion processes within social networks. The new "Logic for Diffusion in Social Networks" is a dynamic extension of standard hybrid logic, allowing to model complex phenomena involving several properties of agents. We provide a complete axiomatization and a terminating and complete tableau system for this logic and show how to apply the framework to diffusion phenomena documented in social networks analysis.

1. Introduction

Social networks are groups of agents structured by some social relationship/links such as family ties, being colleagues, or "following" on social media sites – in other words directed or undirected graphs. In the last decade, the study of social networks, and networks in general, has seen a rapid increase (classical textbooks include [33,26,18]). A variety of aspects of networks have been studied: how they emerge, how they change, what their structural properties are, what social roles they play, etc. This paper focuses on dynamic processes occurring within social networks, such as diffusion of information, viruses, trends, opinions, or behaviors, for instance. What typically characterizes such processes is that their dynamics are local: whether an agent adopts a behavior/opinion/disease/product/trend depends on whether the agents linked to him within the social network have adopted it already. Since social networks are graphs and since the dynamics depend on *local* properties, it seems natural to develop a dynamic *modal* logic to reason about such phenomena in social networks. This is exactly what this paper does.

1.1. Diffusion phenomena

To clarify the type of processes our framework is designed for, let us start by considering an example: the diffusion of a disease within a population. Assume that each agent of the population is in either of

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two states: *infected* with the disease or *susceptible* to it. This type of models is commonly called "the SI Model" in the social networks literature [33]. Moreover, assume that the disease can only be contracted by being physically in contact with an infected agent. Consider the network consisting of agents in a given population, where two agents are linked if they are in contact with each other. (If an agent a is linked to an agent b, we will also call b a "neighbor" of a or a "friend" of a.)

Consider now how such an infection spreads through a (finite) population. This depends on how contagious the disease is. Assume that each agent linked to an infected agent in the network will get infected too at the next moment. This means that if some agent i is infected to start with, all agents directly linked to him will be infected at the next moment, and then all agents linked to the agents linked to him, and so on. Finally, according to this rule of contagion, all agents in (the connected component of) the population will contract the disease after some time. However, note that the social network structure constrains how fast such a disease spreads and what measures would be needed to contain it – the shortest network-path from an agent a to the initially infected agent i determines how long it takes before agent a will get infected. In this example, the dynamics is essentially captured by the following local diffusion rule: If any of your neighbors is infected, become infected yourself at the next moment.

The long term dynamics of such contagion phenomena can also be investigated. Assuming that once an agent gets infected she will stay infected forever, each finite connected component of the network will reach a *stable* state where everybody is infected. However, one could very well imagine a different diffusion rule: after being infected, an agent immediately recovers and becomes susceptible again at the next moment. According to this new dynamic rule, agents might keep alternating forever between being infected and being susceptible and the network might never reach a stable state.

This simple "SI" example can be enriched in several ways. First, the health status of an agent could take more than two values. In the so-called "SIR Model" (*susceptible, infected, recovered*, see for instance [33]), agents can become "recovered" after being infected, which might mean that they have become immune to the disease or that they will move back to being susceptible.

In the above, only the current health status of the agents matters to determine their future health status. A second way to enrich such diffusion models is to take into account other features of agents which might interfere with the diffusion of the infection. For instance, imagine a genetic type such that agents of this type are immune to the disease or stay infected for longer. In this case, the epidemic behavior reveal more complexity, and the diffusion rule needs to combine several properties of agents to take into account all factors.

Two aspects of the above examples will be particularly relevant to this paper. First, agents have certain *properties* such as health status, genetic type, age, gender, hair color, etc. For each agent all these properties are instantiated by particular *features* (or *values*), such as infected, is of the immune genetic type, 34 years old, female, redhead, etc. The features of some of these properties are spreading within the network (in our simple examples, only the health status features are). For each property the associated possible features will come from some fixed set of values, such as: the three possible health states, numbers 1 to 130, male or female, a set of possible hair colors, etc. This assignment of one value to each property will be captured by a particular kind of atomic propositions in the logic developed in the next section.

The second thing to remark is that the dynamics are defined in a purely *local* way. In the above, an agent changes her health status from being susceptible to being infected if at least one of her neighbors is infected. Other kinds of local dynamics could be considered: for instance, dying your hair red if all of your friends have red hair or if at least one of your friends has a friend who has red hair. This type of local conditions is ideally described by formulas of a modal language. Thus, using an extension of basic modal logic will provide a natural way of defining a large variety of dynamic processes on social networks.