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Editorial

Category theory, logic and formal linguistics: Some connections, old and new

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ABSTRACT

We seize the opportunity of the publication of selected papers from the *Logic, categories, semantics* workshop to survey some current trends in logic, namely intuitionistic and linear type theories, that interweave categorical, geometrical and computational considerations. We thereafter present how these rich logical frameworks can model the way language conveys meaning.

1. A seminar and workshop on category theory, logic and linguistic applications

The present issue of the *Journal of Applied Logic* gathers a selection of papers presented at a workshop *Logic, categories, semantics* held in Bordeaux in November 2010. This workshop was organised as a fitting conclusion to the activities of a weekly reading group called *Sheaves in logic and in geometry* in 2009/2010 and *Logic, categories, geometry* in 2010/2011.

Those activities are common to the maths and computing departments of the University of Bordeaux (IMB-CNRS and LaBRI-CNRS). Although he did not contribute to this introduction, we must thank Boas Erez (IMB, Bordeaux) who was an enthusiastic participant and speaker in our reading group. We would like to thank for their scientific and financial support: our two departments, (Institut de Mathématiques de Bordeaux, Laboratoire Bordelais de Recherche en Informatique), INRIA Bordeaux Sud-Ouest, the ANR project LOCI, and the project ITIPY (Aquitaine Region) and the French mathematical society (SMF).

Unfortunately, the material in this issue of the *Journal of Applied Logic* does not cover all the great talks we had the privilege to hear, but one can find published material that more or less cover these talks (and these references themselves include further references):

- Pierre Cartier (IHES, Bures) *Sur les origines de la connexion entre logique intuitionniste, catégories et faisceaux*, cf. [9];
- Jean-Yves Girard (CNRS, IML, Marseille) *Interdire ou réfuter? Le statut ambigu de la normativité*, cf. [19];
- Paul-André Melliès (CNRS, PPS, Paris) *Logical proofs understood as topological knots*, cf. [33];

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- Michael Moortgat (Universiteit Utrecht) *Continuation semantics for generalized Lambek calculi*, cf. [35];
- Carl Pollard (Ohio State University, Columbus) *Remarks on categorical semantics of natural language*, cf. [37].

2. Intuitionistic logic

Type theory and categorical logic, which are at the hear of this issue, all rely on intuitionistic logic. Even linear logic, which is also discussed in two articles of this issue, may be viewed as a refinement of intuitionistic logic.

It is quite a challenge to explain briefly what intuitionistic logic is and how it relates to other logical trends. Regarding the historical development of logic, we refer to [23] and regarding a standard logic master course we refer the reader to the textbook [43] which includes a chapter on intuitionistic logic.

Logic is concerned with "truth", and let us vaguely say that "truth" holds or not of formulae that are formalisations of common language sentences, or of mathematical statements, and as such logic has at least two facets. A formula F can be true because it is correctly derived from formulae that are already established or assumed to be true, and this is the proof theoretical view of truth that started with Aristotle. The same formula F can also be true in all or in some situation once the symbols in the formula are properly interpreted: that's the model theoretical view of truth, which is much more recent. Besides these two well established viewpoints, there exists a slightly different view of "truth" with Ancient origins, dialectics which emerge from the interaction between proofs and refutations. Nowadays this view culminates in game theoretical semantics, and ludics, and the presentations by J.-Y. Girard and P.-A. Melliès during our workshop developed this interactive view [19,33].

An important theorem, due to Gödel as several other fundamental results in logic, is that for first order classical logic, the two notions of truth coincide: a formula is true in any model if and only if it can be proved. This result admits a more striking formulation as: a formula C can be derived from formulae $H_1, \dots, H_n$ if and only if any model satisfying $H_1, \dots, H_n$ satisfies C as well. Such a result holds for classical propositional logic and for classical first-order logic, also known as predicate calculus.

Intuitionistic logic comes from doubts that appeared during the crisis of the foundations of mathematics, between the 19th and 20th centuries. These doubts originate from existential statements in infinite sets. For instance it is admittedly a bit strange, especially in an infinite universe, that $\exists x(B(x) \Rightarrow \forall y B(y))$ holds, although it is absolutely correct in classical logic using principles like $U \Rightarrow V \equiv \neg U \vee V$, $\neg\neg A \equiv A$, $\neg \forall x A(x) \equiv \exists x \neg A(x)$. According to intuitionistic logic, *tertium non datur*, or any of the principles that are equivalent to it, like *reductio ad absurdum* or *Peirce's law*, should be left out from deduction principles.

When restricting ourselves to intuitionistic logic both facets of truth are modified:

- In order to maintain the completeness results, models need to be more complex: the usual models ought to be replaced with structured families of classical models, like Kripke models or (pre)sheaves, cf. Section 3.
- Intuitionistic proofs are more interesting: as opposed to classical proofs, they can be viewed as algorithms (as programs of a typed functional programming language) or as morphisms in a category (which is not a poset, i.e. with many morphisms from an object to another).

Each of these two aspects brings connections with category theory. The first one yields models of higher order intuitionistic logic, e.g. with (pre)sheaves of $\mathcal{L}$-structures (classical models, roughly speaking). The second one yields models that interpret proofs as morphisms in a cartesian closed category.