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A propositional linear time logic with time flow isomorphic to ω^2

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A B S T R A C T

Primarily guided with the idea to express zero-time transitions by means of temporal propositional language, we have developed a temporal logic where the time flow is isomorphic to ordinal ω^2 (concatenation of ω copies of ω). If we think of ω^2 as lexicographically ordered $\omega \times \omega$, then any particular zero-time transition can be represented by states whose indices are all elements of some $\{n\} \times \omega$. In order to express non-infinitesimal transitions, we have introduced a new unary temporal operator $[\omega]$ (ω-jump), whose effect on the time flow is the same as the effect of $\alpha \mapsto \alpha + \omega$ in ω^2. In terms of lexicographically ordered $\omega \times \omega$, $[\omega]\phi$ is satisfied in $\langle i, j\rangle$-th time instant iff ϕ is satisfied in $\langle i+1, 0\rangle$-th time instant. Moreover, in order to formally capture the natural semantics of the until operator $\mathtt{U}$, we have introduced a local variant $\mathtt{u}$ of the until operator. More precisely, $\phi\,\mathtt{u}\,\psi$ is satisfied in $\langle i, j\rangle$-th time instant iff ψ is satisfied in $\langle i, j+k\rangle$-th time instant for some nonnegative integer k, and ϕ is satisfied in $\langle i, j+l\rangle$-th time instant for all $0 \leqslant l < k$. As in many of our previous publications, the leitmotif is the usage of infinitary inference rules in order to achieve the strong completeness.

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1. Introduction

In [17], A. Gargantini, D. Mandrioli and A. Morzenti have proposed a general framework for formal description of systems with so-called zero-time transitions, illustrated through Petri nets as state machines and TRIO as assertion language. The key novelty in their approach of modeling zero-time transitions was introduction of infinitesimals in the time flow. More precisely, they have adopted and operationalized a

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natural assumption that transitions of any particular system from one state to the next are not instantaneous but infinitesimal with respect to the execution time of the entire system. From the logician's point of view, we may see [17] as a skillful application of the interpretation method, particularly if we analyze proofs of the correctness (or adequacy) of proposed modeling. In the recent companion paper [15], L. Ferrucci, D. Mandrioli, A. Morzenti and M. Rossi uses concepts from non-standard analysis and provide notions of micro and macro steps in an extension of the TRIO metric temporal general-purpose specification language. The key difference between that paper and our approach is that we restrict the time flow to a concrete well-ordering. In [7,8] a family of temporal logics that extend LTL to allow time flows isomorphic to any countable limit ordinal are presented. Decidability of those logics is analyzed using generalized Büchi automata. Our logic corresponds to the logic with time isomorphic to ω^2, where the considered operators (in their notation) are $\bigcirc^1$, $\bigcirc^\omega$ and $\mathtt{U}^{\omega^2}$.

As a natural consequence of our research background and scientific taste, which is in large part focused on probability logic, temporal logic and tame fragments of L_{ω_1} and $L_{\omega_1,\omega}$ logics (among those are admissible fragments in Barwise sense, see [1]), inspired by the work presented in [17] we have decided to develop a discrete linear time temporal propositional logic adequate for modeling zero-time transitions. Following the concept of non-instantaneous transitions of a system and discrete linear time model, we end up with the time flow isomorphic to concatenation of ω copies of ω, i.e. with ω^2 as the model of the time flow.

Arguably, the most intuitive representation of the ordinal ω^2 is the lexicographically ordered $\omega \times \omega$. For our purpose, changes of the first coordinate represent different states of a system, while the changes of the second coordinate represent transitions from one state to the next. Hence, it was natural to introduce the following temporal operators:

- $[\omega]$. It represents next state of a system. In the terms of a time flow, it corresponds to the operation $\alpha \mapsto \alpha + \omega$ on ω^2. Semantically, $[\omega]\phi$ is satisfied in the $\langle i, j\rangle$-th time instant iff ϕ is satisfied in the $\langle i+1, 0\rangle$-th time instant;
- $[1]$. It represents the infinitesimal change of a system within some state. In terms of a time flow, it behaves like the usual next operator: $[1]\phi$ is satisfied in the $\langle i, j\rangle$-th moment iff ϕ is satisfied in the $\langle i, j+1\rangle$-th moment;
- $\mathtt{U}$. It represents the adequate generalization of the until operator from ω to ω^2. Semantically, $\phi \mathtt{U} \psi$ is satisfied in the $\langle i, j\rangle$-th moment iff there is $\langle k, l\rangle \geqslant_{\text{lex}} \langle i, j\rangle$ so that ψ is satisfied in the $\langle k, l\rangle$-th moment, and for all $\langle r, s\rangle$ such that $\langle i, j\rangle \leqslant_{\text{lex}} \langle r, s\rangle <_{\text{lex}} \langle k, l\rangle$, ϕ is satisfied in $\langle r, s\rangle$-th moment. Here $\leqslant_{\text{lex}}$ denotes lexicographical ordering;
- $\mathtt{u}$. It is a local version of the until operator. Semantically, $\phi \mathtt{u} \psi$ is satisfied in the $\langle i, j\rangle$-th moment iff there is a nonnegative integer k such that ψ is satisfied in the $\langle i, j+k\rangle$-th moment, and for all $l < k$, ϕ is satisfied in $\langle i, j+l\rangle$-th moment.

The main technical results are the proofs of the completeness theorem and determination of the complexity for the satisfiability procedure (PSPACE).

1.1. Related work

The present paper can be classified as a research related to discrete linear time temporal logics, with particular application on system descriptions and handling zero-time transitions in Petri nets. For modal and temporal part, we refer the reader to [2,4,5,3,6,12,13,11,14,16,18,19,21–23]. The infinitary techniques presented here (application of infinitary inference rules in order to overcome inherited noncompactness) are connected with our previous research, see [10,9,20]. Decidability argumentation presented here is the modification of the work of A. Sistla and E. Clarke presented in [24]. The motivation for this particular

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