Relativized common knowledge for dynamic epistemic logic

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A B S T R A C T

Relativized common knowledge is a generalization of common knowledge proposed for public announcement logic by treating knowledge update as relativization. Among other things relativized common knowledge, unlike standard common knowledge, allows reduction axioms for the public announcement operators. Public announcement logic can be seen as one of the simplest special cases of action model logic (AML). However, so far no notion of relativized common knowledge has been proposed for AML in general. That is what we do in this paper. We propose a notion of action model relativized common knowledge for action model logic, and study expressive power and complete axiomatizations of resulting logics. Along the way we fill some gaps in existing expressivity results for standard relativized common knowledge.

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1. Introduction

Dynamic epistemic logics extend traditional ("static") epistemic logic [6,12] in order to make it possible to express epistemic pre- and post conditions of actions and other events. The two probably most prominent examples are *public announcement logic* [13] in which actions are assumed to be truthful public announcements, and *action model logic* [1,2] which can be used to reason about a very general class of events.

Common knowledge and other types of group knowledge are of key importance in dynamic epistemic logic. For example, events like public announcements are often used to achieve common knowledge.

Relativized common knowledge is a generalization of common knowledge proposed for public announcement logic by treating knowledge update as relativization [14,15]. Relativized common knowledge has a

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particular benefit over standard common knowledge for public announcement logic. It is well known that public announcement logic without common knowledge is not more expressive than (static) epistemic logic (without common knowledge), but public announcement logic *with common knowledge* is strictly more expressive than epistemic logic with common knowledge [2]. However, this turns again when common knowledge is replaced by relativized common knowledge: public announcement logic with relativized common knowledge can be reduced to epistemic logic with relativized common knowledge. This simplifies completeness proofs: by adding reduction axioms the problem of completeness for public announcement logic with relativized common knowledge reduces to the problem of completeness for epistemic logic with relativized common knowledge.

Relativized common knowledge was conceived specifically for public announcement logic: it incorporates the kind of knowledge update that that logic models, namely public announcements. As far as we know, no generalization of relativized common knowledge to arbitrary events have been proposed so far. In this paper we propose a notion of *action model relativized common knowledge*, which is a generalization of relativized common knowledge for action model logic. We study variants of action model logic with a new operator for action model relativized common knowledge, in particular their expressive power and completeness. Like standard relativized common knowledge and public announcement logic, action model relativized common knowledge allows reduction axioms for action model logic. Thus, the problem of completeness reduces to the problem of completeness for epistemic logic with action model relativized common knowledge.

Along the way we fill some gaps in existing expressivity results for standard relativized common knowledge: we strengthen a recent result by Kuijer [11] showing that epistemic logic with (standard) relativized common knowledge is strictly more expressive than epistemic logic with common knowledge for S5 structures, from the case of three or more agents to include the case of two agents, and we prove corresponding results when *distributed knowledge* is added to the languages.

The paper is organized as follows. In the next section we briefly review some background definitions and known results, before introducing action model relativized common knowledge in Section 3. In Section 4 we study expressivity issues. In Section 5 we present axiomatizations of different variants of action model logic with action model relativized common knowledge, and prove completeness. The combination of action model relativized common knowledge and distributed knowledge operators causes additional complications. We end with a discussion in Section 6.

2. Background

We briefly review the key definitions and results from (dynamic) epistemic logic we will build on. We refer the reader to, e.g., [18] for more details.

2.1. Static and dynamic epistemic logics

Let PROP be a countable set of propositional variables and AG a finite set of agent symbols.

Definition 1 *(Action models).* Let $\mathcal{L}$ be a language. $\mathfrak{A}^{\mathcal{L}} = (\mathsf{A}, \simeq, \mathsf{pre}^{\mathfrak{A}^{\mathcal{L}}})$ is called an *action model for $\mathcal{L}$* (or simply an *action for $\mathcal{L}$*, or *$\mathcal{L}$-action*), if the following hold:

- A is a non-empty *finite* set of *action states*, called the *domain* of $\mathfrak{A}^{\mathcal{L}}$;
- $\simeq$: AG $\rightarrow \wp(\mathsf{A} \times \mathsf{A})$ maps every agent a to an equivalence relation $\simeq_a$ on A;
- $\mathsf{pre}^{\mathfrak{A}^{\mathcal{L}}} : \mathsf{A} \rightarrow \mathcal{L}$ is a *precondition function*.