Journal of Applied Logic 11 (2013) 364–372

Contents lists available at SciVerse ScienceDirect

Journal of Applied Logic

www.elsevier.com/locate/jal

Confirmation as partial entailment: A representation theorem in inductive logic

Vincenzo Crupi [a,b,*], Katya Tentori [c,d]

[a] Department of Philosophy and Education, University of Turin, Italy
[b] Munich Center for Mathematical Philosophy, Ludwig Maximilian University, Germany
[c] Department of Cognitive Sciences and Education, University of Trento, Italy
[d] Center for Mind/Brain Sciences, University of Trento, Italy

ARTICLE INFO

Article history:
Available online 18 March 2013

Keywords:
Probability
Confirmation
Inductive logic

ABSTRACT

The most prominent research program in inductive logic – here just labeled *The Program*, for simplicity – relies on probability theory as its main building block and aims at a proper generalization of deductive-logical relations by a theory of partial entailment. We prove a representation theorem by which a class of ordinally equivalent measures of *inductive support* or *confirmation* is singled out as providing a uniquely coherent way to work out these two major sources of inspiration of The Program.

The current state of inductive logic may appear puzzling. Some highly sophisticated observers in philosophy, for instance, have come to see the very term as "slightly antiquated" (see [32, p. 291]). Yet the central issue of inductive logic – i.e., the evaluation of how given premises or data affect the credibility of conclusions or hypotheses of interest – never ceased to play a significant role in a wide range of research domains. Up to recent times, striking examples arise from fields such as cognitive psychology, computer science and the law (by way of illustration, see [19], [2], and [1], respectively). Thus, the *problem* of inductive logic seems not to have lost its relevance, which provides motivation to stick to the label after all, whatever its fate in certain philosophical quarters.

Survey presentations usually agree on one account, i.e., that much contemporary work in inductive logic has consistently relied on two pillars. First, *probability* (in its modern mathematical meaning) is viewed as the main "building block" for inductive-logical theorizing. And second, inductive logic is meant to provide an analogue of classical deductive logic in some suitable sense (see [12] and [21]). For the sake of convenience, we will simply use *The Program* to denote the combination of these two guidelines in inductive logic research.

In this contribution, we do not mean to defend The Program as such. We will rather enrich it through a novel formal result, i.e., a representation theorem by which a class of ordinally equivalent measures of *inductive support* or *confirmation* is singled out as capturing a small number of axioms. These axioms, we will argue, provide an unusually neat instantiation of the spirit of The Program itself.

1. Induction and probability

Broadly speaking, the case for the probabilistic side of The Program is pretty straightforward and runs more or less as follows. It is a platitude that induction arises in the presence of uncertainty, and probability is widely recognized as the formal

* Corresponding author at: Department of Philosophy and Education, University of Turin, via Sant'Ottavio 20, 10124, Turin, Italy.
E-mail address: vincenzo.crupi@unito.it (V. Crupi).

1570-8683/$ – see front matter
http://dx.doi.org/10.1016/j.jal.2013.03.002

representation of uncertainty that is best understood and motivated. (For an updated survey of the main alternative options available, though, see [28].) In order to exploit this point in more detail, we will now need a few technical preliminaries.

Let L be a propositional language. To ensure mathematical definiteness we will focus on the set L_c of the contingent formulas in L (i.e., those expressing neither logical truths nor logical falsehoods) and on the set $\mathbf{P}$ of all *regular* probability functions that can be defined over L, so that for any $\alpha \in L_c$ and $P \in \mathbf{P}$, $0 < P(\alpha) < 1$. Each element $P \in \mathbf{P}$ can now be seen as representing a possible (non-dogmatic, see [26, p. 70]) state of belief concerning a domain described in L. We will posit a function $C : \{L_c \times L_c \times \mathbf{P}\} \to \Re$ as representing the fundamental inductive-logical relation of support or confirmation and adopt the notation $C_P(h, e)$, with $e, h \in L_c$ denoting the premise (or the conjunction of a collection of premises) and the conclusion of an inductive argument, respectively.[1] Our first axiom will then be as follows:

A0 *(Formality).* There exists a function g such that, for any $e, h \in L_c$ and $P \in \mathbf{P}$, $C_P(h, e) = g[P(h \wedge e), P(h), P(e)]$.

Note that the probability distribution over the algebra generated by e and h is entirely determined by $P(h \wedge e)$, $P(h)$ and $P(e)$. Hence A0 simply states that $C_P(h, e)$ depends on that distribution, and nothing else. This is a widespread (although often tacit) assumption in discussions of induction in a probabilistic framework. From Keynes and Carnap onwards, theorists pursuing The Program are bound to subscribe to A0 more or less as a matter of course. When prompted by technical reasons, moreover, inductive logicians working under the heading of Bayesian confirmation theory (or other related labels) have expressed explicit endorsement of it.[2]

Now consider the following:

A1 *(Final probability incrementality).* For any $e_1, e_2, h \in L_c$ and any $P \in \mathbf{P}$, $C_P(h, e_1) \gtreqless C_P(h, e_2)$ iff $P(h|e_1) \gtreqless P(h|e_2)$.

A1 states that, for any conclusion h, inductive support is an increasing function of the posterior probability conditional on the premise (or conjunction of premises) e at issue. To the best of our knowledge, this also counts as virtually unchallenged an assumption in probabilistic analyses of inductive inference.[3] Notably, it already conveys a minimal form of alignment between inductive and deductive logic. For, if violations of A1 are allowed, then one might have cases in which $e_1 \vDash h$ while $e_2 \nvDash h$, so that $P(h|e_1) = 1 > P(h|e_2)$, and yet $C_P(h, e_1) < C_P(h, e_2)$ (see [52, p. 109] for an example). We will now have to tackle this point in a more thorough and general fashion.

2. Partial entailment – taken seriously

What we have called The Program of inductive logic research has been pursued in a number of variants, mostly depending, as James Hawthorne has observed, on "precisely how the deductive model is emulated" [21]. Our current proposal amounts to downright revival of an old and illustrious idea. According to this view, inductive logic should parallel the deductive model by providing a generalized, quantitative theory of *partial entailment*.[4] The following revealing passage, again from [21], attests to the enduring influence of this notion, albeit in a pessimistic vein:

> A collection of premise sentences *logically entails* a conclusion sentence just when the negation of the conclusion is *logically inconsistent* with those premises. An inductive logic must, it seems, deviate from this paradigm [...]. Although the notion of *inductive support* is analogous to the deductive notion of *logical entailment*, and is arguably an extension of it, there seems to be no inductive logic extension of the notion of *logical inconsistency* – at least none that is interdefinable with *inductive support* in the way that *logical inconsistency* is interdefinable with *logical entailment*. (All italics in the original.)

A central goal of our discussion here is to show that this resignation is hasty. It is perfectly possible, we urge, to have a sound inductive-logical extension of the notion of logical inconsistency that is indeed interdefinable with inductive support in essentially the same way that logical inconsistency is interdefinable with logical entailment. So much so, we submit, that one can safely and fruitfully embed into axioms those very properties that inductive logic would inevitably lack according to Hawthorne.

First, we will assume the inductive-logical measure $C_P(h, e)$ to exhibit a commutative behavior whenever e and h are inductively at odds (i.e., negatively correlated), thus paralleling the symmetric nature of logical inconsistency, as follows:

A2 *(Partial inconsistency).* For any $e, h \in L_c$ and any $P \in \mathbf{P}$, if $P(h \wedge e) \leqslant P(h)P(e)$, then $C_P(h, e) = C_P(e, h)$.

[1] To allow for relevant background knowledge and assumptions, a further term B should be included, thus having $C_P(h, e|B)$. Such a term will be omitted from our notation for simple reasons of convenience, as it is inconsequential for our discussion.

[2] See [15, p. 322], [16, pp. 127–128], and [36, p. 21]. The label *formality* is taken from [53,54].

[3] Relevant occurrences of A1 or closely related principles include the following: [5, pp. 77–80], [8, p. 670], [10, p. 58], [13, p. 506], [17, p. 295], [20, p. 122], [22], [25, p. 53], [50, pp. 219–221], and [51, p. 60].

[4] The idea of partial entailment can be shown to reach back to [31] and [3]. For the label, however, [44] is a key reference.