Journal of Applied Logic 11 (2013) 421–429

Contents lists available at SciVerse ScienceDirect

Journal of Applied Logic

www.elsevier.com/locate/jal

An observation on Carnap's Continuum and stochastic independencies

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ARTICLE INFO

Article history:
Available online 18 March 2013

Keywords:
Carnap's Continuum
Atom Exchangeability
Stochastic independence
Inductive Logic

ABSTRACT

We characterize those identities and independencies which hold for all probability functions on a unary language satisfying the Principle of Atom Exchangeability. We then show that if this is strengthen to the requirement that Johnson's Sufficientness Principle holds, thus giving Carnap's Continuum of inductive methods for languages with at least two predicates, then new and somewhat inexplicable identities and independencies emerge, the latter even in the case of Carnap's Continuum for the language with just a single predicate.

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1. Introduction

To this day the *Continuum of Inductive Methods* described by Carnap in [1–4] continues to be adapted and promoted as paradigm solutions to various problems within Inductive Logic. For example arithmetic combinations of these functions figure almost exclusively in recent attempts to provide probability functions exhibiting certain specific features of analogical influence, see [5,11,12,15,16].

There seem to be several good reasons for this focus. Firstly this Continuum has a widely acceptable justification in terms of its 'rationality': There is a putatively rational requirement, namely Johnson's Sufficientness Principle, that we can impose on an inductive method (i.e. probability function) which forces it to be precisely a member of Carnap's Continuum (see also Johnson's earlier derivation of this in [9]), at least when we assume that the language has more than one predicate. Secondly the Continuum has a simple form, making it easy to work with, whilst the parameter it involves has a clear interpretation which readily permits generalizations.

Carnap's original goal in his Inductive Logic programme was to develop an inductive method which could be applied to real world problems of induction, or more generally the assignment of probabilities based on some finite body of evidence, and which furthermore was *logical* in the sense that it's conclusions followed mechanically from the evidence via certain precisely formulated rules or principles. The arrival on the scene of Goodman's Grue Paradox, [6,7], however highlighted an evident flaw in the *practicality* of the approach; that in real (as opposed to toy) examples there is usually so much available evidence that even if it could be suitably formulated in the language of the problem it would be completely infeasible to take it as one's premise set.

Whilst many philosophers have seen this as the end of the programme as a practical, rather than simply a theoretical, project, nevertheless apparently similar aspirations to Carnap's still seem to underlie papers such as those on analogical reasoning cited above. One explanation for this is that whilst *all* our available knowledge in a real world situation is just too much to handle nevertheless most of it should be redundant or irrelevant and possibly what really does matter *can* be simply formulated. This raises the question we shall consider in this paper, to what extent is this a reasonable assumption for the members of Carnap's Continuum, more precisely under what circumstances is a sentence θ stochastically independent of a sentence ϕ for all members of Carnap's Continuum?

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http://dx.doi.org/10.1016/j.jal.2013.03.004

Before that however we need to spend a little time introducing some standard notation. The experienced reader might therefore be advised to skip the next section, only referring back to it as necessary.

2. Notation

Let L be a predicate language with just q (unary) predicates, $P_1, P_2, \ldots, P_q$, constants a_1 for $i = 1, 2, 3, \ldots$ and no other relation, constant or function symbols. As usual the intention here is that these a_i exhaust the universe.

Let $\alpha_1(x), \alpha_2(x), \ldots, \alpha_{2^n}(x)$ denote the *atoms* of L, that is the 2^q formulae of L of the form

$$\pm P_1(x) \wedge \pm P_2(x) \wedge \cdots \wedge \pm P_q(x).$$

So for example the atoms in the case $q = 2$ are $P_1(x) \wedge P_2(x)$, $P_1(x) \wedge \neg P_2(x)$, $\neg P_1(x) \wedge P_2(x)$, $\neg P_1(x) \wedge \neg P_2(x)$. Knowing which atom an a_i satisfies tells us exactly which of the $P_j(x)$ a_i does or does not satisfy, and hence tells us everything there is to know about a_i.

A *state description*, $\Theta(b_1, b_2, \ldots, b_m)$, for distinct choices $b_1, b_2, \ldots, b_m$ from the a_i, is a sentence of the form

$$\bigwedge_{i=1}^{m} \alpha_{j_i}(b_i), \tag{1}$$

and similarly tells us all there is to know about $b_1, b_2, \ldots, b_m$.

Notice that the state descriptions for $b_1, b_2, \ldots, b_m$ are disjoint and any quantifier free sentence $\phi(b_1, b_2, \ldots, b_m)$ of L is logically equivalent to a disjunction

$$\bigvee_{k=1}^{s} \Theta_k(b_1, b_2, \ldots, b_m)$$

of distinct state descriptions $\Theta_k(\vec{b})$ for $b_1, b_2, \ldots, b_m$. Hence if w is a probability function on L (for a definition see for example [8] or [14]) then

$$w\big(\phi(b_1, b_2, \ldots, b_m)\big) = \sum_{k=1}^{s} w\big(\Theta_k(b_1, b_2, \ldots, b_m)\big). \tag{2}$$

We say that w satisfies *Constant Exchangeability, Ex*, if $w(\phi(b_1, b_2, \ldots, b_m))$ depends only on $\phi(x_1, x_2, \ldots, x_m)$ and not on the (distinct) instantiating constants $b_1, b_2, \ldots, b_m$. By (2) it is already enough that this holds for state descriptions. Since all the probability functions we shall consider will satisfy Ex our results will apply for general $b_1, b_2, \ldots, b_m$ once proven for $a_1, a_2, \ldots, a_m$.

The *spectrum* of a state description $\Theta(b_1, \ldots, b_m)$ as in (1) is the multiset[1] $\bar{n} = \{n_1, n_2, \ldots, n_{2^q}\}$, where n_i is the number of times that the atom $\alpha_i(x)$ appears amongst the $\alpha_{j_1}(x), \alpha_{j_2}(x), \ldots, \alpha_{j_m}(x)$.

We say that w satisfies *Atom Exchangeability*, *Ax*, if $w(\Theta(b_1, b_2, \ldots, b_m))$ depends only on the spectrum $\bar{n}$ of the state description $\Theta(b_1, b_2, \ldots, b_m)$. In this case we shall write $w(\bar{n})$ for $w(\Theta(\vec{b}))$.

Finally we say that w satisfies *Johnson's Sufficientness Principle, JSP*, if for a state description $\Theta(b_1, b_2, \ldots, b_m)$ as in (1), $w(\alpha_i(b_{m+1}) \mid \Theta(b_1, b_2, \ldots, b_m))$ depends only on m and n_i. It is well known that JSP implies Ax which in turn implies Ex.

As shown originally by Johnson, [9] (and independently later by Kemeny, see [4, section 19] and [10]) if the number of predicates, q, is at least 2 and the probability function w satisfies JSP then w is a member of *Carnap's Continuum of Inductive Methods*. That is, $w = c_\lambda$ for some $0 \leqslant \lambda \leqslant \infty$ where, with the above notation, c_λ is the probability function satisfying Ax such that

$$c_\lambda\big(\alpha_i(b_{m+1}) \wedge \Theta(b_1, b_2, \ldots, b_m)\big) = \frac{(n_i + \lambda/2^q)}{(m + \lambda)} \cdot c_\lambda\big(\Theta(b_1, b_2, \ldots, b_m)\big). \tag{3}$$

The cases $\lambda = 0, \infty$ here are rather exceptional and until further notice we shall restrict ourselves to $0 < \lambda < \infty$ when discussing the c_λ (though still referring to these as Carnap's Continuum).

3. Stochastic independence and Ax

Let w be a probability function on L satisfying Ax. Then from (2) for $\phi(a_1, \ldots, a_m)$ a sentence of L,

$$w\big(\phi(a_1, \ldots, a_m)\big) = \sum_{\bar{n}} f_\phi(\bar{n}) w(\bar{n}),$$

[1] Multisets are just like sets except that elements may be repeated.

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