Journal of Applied Logic 11 (2013) 487–504

Contents lists available at ScienceDirect

Journal of Applied Logic

www.elsevier.com/locate/jal

Infinity and verifiability in Carnap's inductive logic

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ARTICLE INFO

Article history:
Received 14 May 2013
Accepted 29 July 2013
Available online 20 August 2013

Keywords:
Carnap
Infinity
Truth
Probability
Inductive logic
Extended truth
Extended probability
Constructive
Factual
Empirical

ABSTRACT

Truth of sentences in infinity is discussed in the framework of Rudolf Carnap's inductive logic, which uses finite state descriptions and an asymptotic limit approach for defining probabilities in infinity. This means that Carnap's approach suits well for a semantics which is based on finite observability. However, a proper link between asymptotic probability and truth in infinity is missing from Carnap's treatment. A novel notion of truth in infinity is introduced and referred to as the extended truth. The idea is that the truth of the sentence S is extended by a particular sequence of state descriptions (where the larger one contains all of the smaller ones) iff S is true in each state description of the sequence. The corresponding notion of extended probability is introduced. Some important results are proved for extended truth and extended probability.

1. Introduction and background

There are many interpretations of constructive or intuitionist semantics (e.g. [7,12,24]), but one feature is common to all of them: there is no truth beyond knowledge (verification) and a sentence can only be true if it is knowable (verifiable).

The problem in constructive semantics has been how to define a verification condition for sentences that can neither be mathematically proved nor verified by observations, like universal generalizations in infinite non-mathematical domains of individuals. It follows that constructive semantics has thus far lacked an adequate concept of truth in infinity concerning factual (or empirical, non-mathematical) sentences.

One consequence of this problem is the difficulty to incorporate inductive reasoning in constructive semantics. Inductive reasoning deals with empirical statements that cannot be verified by observation. A paradigmatic example of such a statement is "all ravens are black", because there may always be an unobserved non-black raven. More generally, scientific generalizations like the laws of physics are intended to hold for an infinite number of cases, which renders them non-verifiable by observation.

Probabilistic induction is about assigning probabilities to sentences that cannot be verified by observation. For probabilistic induction concerning statements in an infinite domain of individuals, one needs a concept of probability in infinity, which is usually explained as the probability of the (empirical) truth of the statements in infinity. Hence, for probabilistic induction one needs a definition of empirical truth in infinity, and for constructive probabilistic induction one needs a notion of constructive empirical truth in infinity.

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1570-8683/$ – see front matter
http://dx.doi.org/10.1016/j.jal.2013.07.006

In what follows, the notion of constructive truth will not be defined in any previously existing formal system or by using the meaning explanations pertaining to such a system. A precise concept of constructive empirical truth and probability would be impossible to start with since the very objective of the study is to define such concepts.

However, constructive functions are assumed to be effectively computable – although a reference to a more relaxed concept of the Markov-constructive limit is provided in Section 5.1. Finally, the standard BHK interpretation introduced by Heyting [12] for logical constants will be used unless stated otherwise.

1.1. Constructive empirical truth

The previous literature on constructive empirical truth is scarce. Benenson [1] focuses on an anti-realist (i.e., constructive) interpretation of probability statements themselves (i.e., not sentences to which probabilities are assigned) and holds that "logical relation theories" by Carnap [6] and Keynes [18] (i.e., inductive logic) provide a foundation for an anti-realist explanation of the meaning of probability statements. Prawitz [27], in turn, remains quite skeptical about the possibility for an adequate constructive meaning theory for empirical sentences.

Benenson maintains that a realist account of probabilities is tantamount to an empirical account (for example, frequentist or propensity). Non-empirical interpretations of probability (like epistemic interpretations) are, however, not necessarily anti-realist since probabilities may still exist without being known. As Grove et al. [11, pp. 252–253, 264–273] have pointed out extending the results of Liogon'kii [21], inductive probabilities are in general undecidable, and thus in general not always knowable. However, although inductive probabilities are not necessarily anti-realist, one can say that inductive logic *enables* the definition of an anti-realist concept of probability.

With respect to constructive empirical truth itself, Benenson [1, pp. 57–58] relies on Dummett's remark about the assertibility conditions for empirical statements. According to Dummett, for empirical statements "there will, for the anti-realist, be no question of there being anything in virtue of which they are (definitively) true, but only of things in virtue of which they are probably true". Dummett continues: "[...] and there is nothing to prevent a statement being so used that we do not treat anything as conclusively verifying it." [7, p. 162] Dummett thus holds that no concept of conclusive verification is even needed for empirical statements. The key for Dummett is justified assertibility; a sentence may be justifiably asserted if evidence supporting it is obtained. This evidence does not have to verify the sentence conclusively, which entails that a justifiably assertible sentence may in the future not be justifiably assertible. On this view, the meaning of a sentence is defined by the condition of its justifiable assertibility, not verifiability.

The argumentation thus goes as follows. Justified assertibility is connected with the notion of probability. Evidence increases the probability of a statement. A statement having at least a certain probability can be considered as justifiably assertible.

Hence, evidence can make a sentence more probable and thus justifiably assertible. The problem is that what probability itself means must still be explained. Clearly the probability of a sentence means probability that the sentence holds, i.e., is true. If the domain of discourse – the domain of individuals – is infinite, then truth here means truth in infinity. Hence, it seems difficult to do away with the concept of constructive truth in infinity by replacing verifiability with justified assertibility. Hence, what "probably true" means in constructive terms needs to be defined, but neither Dummett [7] nor Benenson [1] provides an account of this.

1.2. Probability in infinity

It is obvious that, both in classical and constructive semantics, for probabilistic induction concerning sentences of a formal language in an infinite domain of individuals, one needs a concept of probability in infinity. The most obvious approach is to define probability measures directly for formal models of a first-order language with denumerable and infinite domains of individuals, i.e. domains having the cardinality of $\mathbb{N}$, where the classical works include Gaifman [10], Scott and Krauss [29] and Fenstad [9].

Since the concept of truth should be paired with the concept of probability in the sense that the probability of a sentence in infinity is, at least in some sense, interpreted as the probability of its truth in infinity, infinite domains need to be coupled with a notion of truth in an infinite domain. In the case of empirical statements, i.e. sentences whose truth can only be determined by observation, this is not a straightforward matter.

There are basically two options. The first one is that empirical truth is considered as independent of human cognitive capabilities. Sentences are or are not true, but which of these really is the case is not always verifiable even in principle if the actual world is infinite – like for example some contingently true general sentence attributing a property to all individuals, such as $(\forall x)A(x)$ where $A(x)$ is an atomic predicate.

The other option is to take into account what can be said on the basis of finitely many observations. The question is then not whether the sentence is true somehow "out there", beyond our recognition, but rather whether it will be (somehow) confirmed by our observation process.

The first interpretation commits to the existence of an infinite world which transcends human recognition. It also commits to the meaningfulness of saying that any sentence is made true or false by such a world even if the properties of the world are not completely knowable even in principle, and hence there are sentences which cannot be verified or falsified.