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An epistemic and dynamic approach to abductive reasoning: Abductive problem and abductive solution

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ABSTRACT

We propose a study of abductive reasoning addressing it as an epistemic process that involves both an agent's information and the actions that modify this information. More precisely, we present and discuss definitions of an abductive problem and an abductive solution in terms of an agent's information, that is, in terms of knowledge and beliefs. The discussion is then formalised by 'implementing' our definitions in a *dynamic epistemic logic* framework, where the properties of these definitions are studied, an epistemic action that represents the application of an abductive step is introduced, and an illustrative example is provided. A number of the most interesting properties of abductive reasoning (those highlighted by Peirce) are shown to be better modelled within this approach.

1. Introduction

Introduced to modern logic by Charles S. Peirce, abductive reasoning [30,22,25,3] is typically understood as the process of *looking for an explanation for a surprising observation*. Many forms of intellectual tasks, such as medical and fault diagnosis, scientific discovery, legal reasoning, and natural language understanding, belong to this category, thus making abduction one of the most important reasoning processes.

Abductive reasoning has been studied mainly from a purely syntactic perspective. Typical definitions of an abductive problem and its solution(s) are given in terms of a theory and a formula, and therefore most of the work on the subject has focused on: (1) discussing what a theory and a formula should satisfy in order to form an abductive problem, and what a formula should satisfy in order to be an abductive solution [3]; (2) proposing algorithms to find abductive solutions [19,27,28,33,20]; and (3) analysing the structural properties of abductive consequence relations [23,2,41]. In all these studies, which follow the so-called AKM-schema of abduction, certain explanationism and consequentialism are considered, but the epistemic character of abductive reasoning seems to have been pushed into the background. In contrast, the GW-schema [11,42] is based on the concept of *ignorance problem*, which arises when a cognitive agent has a cognitive target that cannot be attained from what she currently knows, thus highlighting the distinctive epistemic feature of abduction that is crucial to our considerations.

In this paper, an *epistemic and dynamic* approach to abductive reasoning is proposed. In this sense, our approach is closer to the ideas in [21,7,1,26] in that it stresses the key role that agents and actions play within the abductive reasoning scenario. Even so, our approach goes one step further, as it fully adopts a dynamic perspective by making explicit the actions involved in the abductive process. We argue that abductive reasoning can be better understood as a *process* that involves *an agent's information*, and with this in mind, not only are definitions of an abductive problem and an abductive

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solution presented and discussed in terms of an agent's knowledge and her beliefs, but a method through which the chosen abductive solution is integrated into the agent's information is also outlined.

More precisely, we propose that, *for a given agent*, an abductive problem arises when she comes to *know or believe* certain χ that *she could not have come to know or believe* with the rest of her information, and that a given η is an abductive solution to this problem *for the agent* whenever, together with the rest of her information, it would have allowed her to (come to) know or believe χ. Then, considering the non-monotonic nature of abductive reasoning, we propose that abductive solutions should be integrated into the agent's information not as knowledge, but rather as beliefs. In a single sentence: we propose that abductive reasoning can be understood as a process of *belief change* that is triggered by an observation and guided by the *knowledge and beliefs* the agent has the ability to derive (Section 2).[1]

After proposing our definitions, we use *dynamic epistemic logic* (*DEL*) tools [39,37] in order to 'implement' our definitions and formalise the discussion (Section 3 introduces the tools and Section 4 formalises our definitions). We show how the classification of abductive problems and abductive solutions proposed in [3] can be extended in this setting, and a number of the properties of these notions are discussed. An example illustrating the way our proposed ideas work is then provided. The paper is completed (Section 5) with a brief summary of the key points of our approach and further directions for research.

Single-agent setting vs multi-agent setting It should be mentioned here that, following the abductive reasoning tradition, and in order to allow an in-depth discussion of our ideas, the present analysis focuses only on the single-agent case: the definitions provided in Section 2 and formalised in Section 4 are given in terms of the knowledge and beliefs of a single agent. Accordingly, the actions considered in this paper, observation (i.e., public announcement) and (public) belief revision, publicly affect the information of our agent. However, a multi-agent setting would allow us to represent richer notions and interactions among agents. In particular, we would be able to consider: (i) definitions of an abductive problem and an abductive solution in terms of the knowledge and beliefs of several agents and, more interestingly, in terms what a set of agents commonly know and/or believe; and (ii) actions that affect the knowledge and beliefs of diverse agents in different ways (e.g., private observations).

2. An epistemic and dynamic perspective

Peirce presents the main features of abduction with the following formulation (see [17]):

> *The surprising fact, C, is observed.*
> *But if A were true, C would be a matter of course.*
> *Hence, there is reason to suspect that A is true.*

And he adds immediately that

> *A cannot be abductively inferred, or if you prefer the expression, cannot be abductively conjectured, until its entire content is already present in the premises, 'If A were true, C would be a matter of course'.*[2]

According to these ideas, abduction is a process that is triggered when a surprising fact is observed by an epistemic agent. After such process ends, an explicative hypothesis is obtained, although the genuine result of the abductive inference is the plausibility of the truth of such hypothesis: in order for the solution to be admissible, it must be verifiable via experimentation.

We use "experimental verification" in a broad sense. Peirce himself discusses several naive conceptions of the notion, which constitute the backbone of philosophical pragmatism. He also points out (see [18]) the particular form of an abductive syllogism by means of the schema *rule* + *result* = *case*, and gives illustrative examples, such as the case of Napoleon: as there are documents and monuments about him, though we have not seen him, we suppose he really existed. Another is the case of fossils: if we find fossils of fish inland, we can conclude that this area was once covered by water.

The truth of the obtained hypothesis is thereby plausibly conjectured. Abduction is therefore an inferential process of epistemic character whose conclusion is rather a provisional proposal that could be revised in the light of new information: abduction is non-monotonic. In [42], another linguistic feature is considered: that the connection between the truth of the hypothesis and the observation is subjunctive, which stresses the necessity of considering the role of an agent in the process.

The present work proposes an approach to abductive reasoning from an epistemic and dynamic perspective. Instead of understanding abductive reasoning as a process that *modifies a theory* whenever *there is a formula* that *is not entailed by*

[1] Diverse forms of abductive reasoning occur in different contexts. The concept has been discussed in various fields, from logic to artificial intelligence and philosophy of science, and this has led to different ideas of what abduction should consist of (see [10], among others). The present work focuses on the simple understanding of the abductive process stated in the opening sentence. Nevertheless, similar epistemic approaches can be made to other interpretations of abduction, as those that involve the creation of new concepts and/or changes in awareness.

[2] [18, p. 231].