Journal of Applied Logic 11 (2013) 147–168

Contents lists available at SciVerse ScienceDirect

Journal of Applied Logic

www.elsevier.com/locate/jal

Two adaptive logics of norm-propositions

Mathieu Beirlaen *, Christian Straßer

Centre for Logic and Philosophy of Science, Ghent University, Blandijnberg 2, 9000 Ghent, Belgium

ARTICLE INFO

Article history:
Available online 9 February 2013

Keywords:
Deontic logic
Normative gaps
Normative conflicts
Adaptive logic
Defeasible reasoning

ABSTRACT

We present two defeasible logics of norm-propositions (statements about norms) that (i) consistently allow for the possibility of normative gaps and normative conflicts, and (ii) map each premise set to a sufficiently rich consequence set. In order to meet (i), we define the logic **LNP**, a conflict- and gap-tolerant logic of norm-propositions capable of formalizing both normative conflicts and normative gaps within the object language. Next, we strengthen **LNP** within the adaptive logic framework for non-monotonic reasoning in order to meet (ii). This results in the adaptive logics $\mathbf{LNP^r}$ and $\mathbf{LNP^m}$, which interpret a given set of premises in such a way that normative conflicts and normative gaps are avoided 'whenever possible'. $\mathbf{LNP^r}$ and $\mathbf{LNP^m}$ are equipped with a preferential semantics and a dynamic proof theory.

1. Introduction

In this section, we introduce in an informal way the key concepts studied in this paper. Formally precise characterizations follow later on.

1.1. Normative conflicts and normative gaps

Ideally, sets of norms issued by agents, authorities, legislators, etc. are both consistent and complete. In our everyday practice, however, such sets often contain normative conflicts and normative gaps. A *normative conflict* occurs when two or more norms are issued that are mutually unsatisfiable. The existence of such conflicts is motivated as follows by Alchourrón and Bulygin:

> Even one and the same authority may command that *p* and that *not p* at the same time, especially when a great number of norms are enacted on the same occasion. This happens when the legislature enacts a very extensive statute, e.g. a Civil Code, that usually contains four to six thousand dispositions. All of them are regarded as promulgated at the same time, by the same authority, so that there is no wonder that they sometimes contain a certain amount of explicit or implicit contradictions [3, pp. 112–113].

Normative conflicts do not always consist of conflicting commands or obligations. They also arise where both an obligation to do something and a (positive) permission not to do it are promulgated [1,3,14,49].

The adaptive logics presented in this paper are able to adequately deal with both normative conflicts and normative gaps. We say that a set of norms contains a *normative gap* with respect to *A* if *A* is neither positively permitted nor forbidden nor obliged. For a defense of the existence of normative gaps, see e.g. [2, Chapters 7, 8], [15].

* Corresponding author. Tel.: +32 9 264 39 79; fax: +32 9 264 41 87.
E-mail addresses: mathieu.beirlaen@UGent.be (M. Beirlaen), christian.strasser@UGent.be (C. Straßer).

1570-8683/$ – see front matter
http://dx.doi.org/10.1016/j.jal.2013.02.001

Note that the formulation refers to *positive permissions* (also, *strong permissions*), i.e. permissions that are either explicitly stated as such, or permissions that are derivable from other explicitly stated permissions or obligations. This is to be distinguished from so-called *weak* or *negative permissions*: A is weakly permitted in case A is not forbidden. Would we replace "positive permission" by "weak permission" in the definition of normative gaps then the concept would be vacuous since each A is either forbidden or not forbidden (and hence, weakly permitted).

The practical use of the distinction between positive and negative permission can be illustrated by means of the legal principle *nullum crimen sine lege*. According to this principle anything which is not forbidden is permitted.[1] Alternatively, the principle states that a negative permission to do A implies a positive permission to do A. Typically, the nullum crimen principle is understood as a rule of closure permitting all the actions not prohibited by penal law [2, pp. 142–143]. We return to this principle in Section 2.1.

We will in the remainder of the paper tacitly assume that in case A is obliged then A is positively permitted. In this case, there is a normative gap with respect to A iff A is neither positively permitted nor forbidden.

Another way to think about normative gaps is in terms of normative determination: A is *normatively determined* if and only if A is either positively permitted or forbidden, which is to say that there is no normative gap with respect to A.[2] We say that a set of norms is *normatively complete* if all of its norms are normatively determined, i.e. if there are no gaps with respect to any of its norms. From the existence of incomplete legal systems, Bulygin concludes that legal gaps are perfectly possible:

> It is not true that all legal systems are necessarily complete. The problem of completeness is an empirical, contingent, question, whose truth depends on the contents of the system. So legal gaps due to the silence of the law (...) are perfectly possible [15, p. 28].

1.2. Norm-propositions and their formal representation

In ordinary language, normative sentences exhibit a characteristic ambiguity. The very same words may be used to enunciate a norm (give a prescription) and to make a normative statement (description) [47, pp. 104–106]. In deontic logic, it is important to carefully distinguish between this prescriptive and descriptive use of norms.

When interpreted prescriptively, a formula of the form $\ulcorner \mathsf{O}A \urcorner$ means something like "you ought to do $\ulcorner A \urcorner$", or "it ought to be that $\ulcorner A \urcorner$", and a formula of the form $\ulcorner \mathsf{P}A \urcorner$ means something like "you may do $\ulcorner A \urcorner$", or "it is permitted that $\ulcorner A \urcorner$".[3] When interpreted descriptively, a formula of the form $\ulcorner \mathsf{O}A \urcorner$ [$\ulcorner \mathsf{P}A \urcorner$] means something like "there is a norm to the effect that $\ulcorner A \urcorner$ is obligatory [permitted]". Thus, in our descriptive reading a formula $\ulcorner \mathsf{P}A \urcorner$ always denotes a strong permission. Following [47], we take the term *norm* to denote the prescriptive, and *norm-proposition* to denote the descriptive interpretation of normative statements.[4]

According to Alchourrón and Bulygin [1–3], any perceived harmony between norms and norm-propositions in deontic logic is merely apparent. Instead of using the same calculus of deontic logic for reasoning with both norms and norm-propositions, we need two separate logics: a logic of norms and a logic of norm-propositions. This paper is concerned with the characterization of a logic of norm-propositions.

In formal language normative conflicts are expressed by formulas such as $\ulcorner \mathsf{O}A \wedge \mathsf{O}\ \textit{not}\ A \urcorner$ in case two obligations conflict, and $\ulcorner \mathsf{O}A \wedge \mathsf{P}\ \textit{not}\ A \urcorner$ in case an obligation conflicts with a permission. We call a conflict of the former kind an OO-*conflict*, and a conflict of the latter kind an OP-*conflict*.

Normative gaps occur if neither $\ulcorner \mathsf{P}A \urcorner$ nor $\ulcorner \mathsf{O}\ \textit{not}\ A \urcorner$ is the case. A full formal characterization of normative gaps is presented after the definition of our formal language. As pointed out above, the permission in question is a strong permission. Weak permissions may be simply defined as the modal dual to O: by $\ulcorner \textit{not}\ \mathsf{O}\ \textit{not}\ A \urcorner$. The latter expresses that "there is no norm to the effect that $\ulcorner \textit{not}\ A \urcorner$ is obliged" and hence it expresses the descriptive meaning of a weak permission. However, we need an independent permission operator P in order to express strong permissions. From $\ulcorner \mathsf{P}A \urcorner$ we cannot infer $\ulcorner \textit{not}\ \mathsf{O}\ \textit{not}\ A \urcorner$ due to the possible existence of an OP-conflict. Similarly we cannot, vice versa, infer $\ulcorner \mathsf{P}A \urcorner$ from $\ulcorner \textit{not}\ \mathsf{O}\ \textit{not}\ A \urcorner$ since, despite the absence of a norm that expresses that $\ulcorner \textit{not}\ A \urcorner$ is obliged, $\ulcorner A \urcorner$ may not be positively permitted.[5]

In the remainder we show how each of the concepts presented in this introductory section is formalized and treated by the logics defined later on in this paper.

---

1 Legal philosophers also refer to this principle as the *sealing legal principle*. We thank an anonymous referee for pointing this out.

2 The notion of normative determination is adopted from [48].

3 Until our formal language is defined, we use brackets "⌜" and "⌝" for denoting formulas.

4 Von Wright [47] and Åqvist [6] cite Ingemar Hedenius as the first philosopher to note the distinction between norms and norm-propositions. According to Hedenius, norms are "genuine", and norm-propositions are "spurious" deontic sentences [25]. The distinction between norms and norm-propositions was later also drawn – among others – by Wedberg [50], Stenius [41], Alchourrón [1], and Hansson [23] (see also [6]).

5 See [2,47] for further arguments against the equivalence of $\ulcorner \mathsf{P}A \urcorner$ and $\ulcorner \textit{not}\ \mathsf{O}\ \textit{not}\ A \urcorner$ in a descriptive setting.