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Simulative belief logic ☆

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ABSTRACT

The ability of ascribing beliefs to others is crucial for human beings to explain and understand each other. Belief ascription has been studied intensively in philosophy and cognitive science. In this paper, we propose a formal framework for belief ascription by simulation. An agent first acquires information about another agent's beliefs by communication. She then inputs the information into her own belief system to generate more beliefs, which she will ascribe to the other agent. In this way, the agent uses her own as a model of others. We present a modal belief logic, which contains private announcement operators for agents' communication, and simulative belief operators for beliefs ascribed to others. We give a complete axiomatic system for the logic.

1. Introduction

The notion of belief is one of the core concepts in philosophy, and has been an active research topic in formal logic since the twentieth century. The standard (modal) approach to characterizing belief goes back to Hintikka [14], whereafter a lot of work on modal belief logics has been done in the areas of philosophical logic, artificial intelligence, and computer science (see [9] and [16] for an overview).

The focus of this paper is on the less-studied topic of formal frameworks for belief ascription. The ability to mutually reason about each other is crucial in our everyday life. In the real world, one usually does not have complete information about others. However, even with incomplete information, one can still reason about others by simulating others' beliefs and ascribing such artificial beliefs to them. By simulation, we mean that one uses oneself as a model of others, and generates simulative beliefs according to one's own belief status and the information one possesses about others.

Belief ascription is currently an active topic in philosophy and cognitive science. However, it still lacks a uniform formal characterization. A default rule has been suggested in [8] for the rule of simulative inference. A type of enclosed inference mechanisms was used in [15]. Simulative inference in a temporal framework was discussed in [11]. An informal framework and an implemented system based on simulative reasoning, called ATT-Meta, was introduced in [6]. A recently published paper [3] examined belief ascription under bounded resources, also within a temporal structure. All of these adopted syntactical representations of belief, in which belief operators were treated as predicates, and belief statements were formalized by first-order sentences. The semantic approach to belief based on possible world models has attracted much more attention than the syntactic approach. It is natural, therefore, to pursue a clear semantics of simulative belief in accordance with this approach.

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The rest of the paper is structured as follows. The next section gives the motivation and the definition of the logic. Section 3 discusses various properties of the logic. Section 4 gives its axiomatization. Related work is discussed in Section 5. We conclude the paper by indicating some future research.

2. Motivation and semantics

The alphabet of simulative belief logic (SBL) consists of a finite non-empty set of agents $Agt = \{1, \ldots, n\}$, a countable set of propositional variables $P = \{p_1, p_2, \ldots\}$, Boolean connectives $\neg$ and $\rightarrow$, a belief operator B_i for each $i \in Agt$, a public belief operator B_i^{Pu} for each $i \in Agt$, a simulative belief operator B_i^j for each pair $i, j \in Agt$ with $i \neq j$, and a communication operator $[tells(j, i, \psi)]$ for each pair $i, j \in Agt$ with $i \neq j$ and each formula ψ. Let φ, ψ, and χ range over formulas. The set $\mathcal{L}$ of formulas is constructed as follows:

$$\varphi =:: p \mid \neg\varphi \mid \varphi \rightarrow \psi \mid B_i\varphi \mid B_i^{Pu}\varphi \mid B_i^j\varphi \mid \big[tells(j, i, \psi)\big]\varphi.$$

Other Boolean connectives are defined as usual. Let

$$\big\langle tells(j, i, \psi)\big\rangle\varphi =_{df} \neg\big[tells(j, i, \psi)\big]\neg\varphi.$$

Meanings of these modal operators will be explained shortly.

The process of belief ascription by simulation can be shown in a simple example. Alice believes that a meteorological office is publicly trustworthy. Through communication, Alice obtains the information that Bob believes the meteorological office has announced that it will rain tomorrow. Alice then concludes that Bob believes that it will rain tomorrow. “It will rain tomorrow” is a belief ascribed to Bob by Alice. It is possible that this belief is not a real belief of either Alice or Bob. Bob may not believe it because he may not trust the meteorological office, as Alice does. Alice may not believe it because she may not hear the weather forecast herself.

In this example, Alice uses her own belief (that the meteorological office is trustworthy) to simulate Bob’s. She possesses a belief of Bob (that there is the weather forecast), and incorporates it into her own belief system. Then she ascribes the outcome (that it will rain tomorrow) to Bob.

Communication is indispensable in such a process. Alice (the simulator) obtains the initial information about Bob (the simulatee) through communication, which allows her to independently develop simulative beliefs that she will ascribe to Bob. In this paper, we assume that agents are sincere and communication is error free. Thus, j tells i that ψ only in the condition that ψ is an actual belief of j.

Dynamic epistemic logic (DEL) has heretofore been the most successful approach to reasoning about knowledge, belief, and communication [21]. Announcement operators like $[\psi \rightarrow G]$ are used to characterize epistemic dynamics. The statement $[\psi \rightarrow G]\varphi$ means that φ is true after ψ is communicated to those agents in G. For the purpose of this paper, we use a type of private announcement operator in the form $[tells(j, i, \psi)]$, with the intended meaning that agent j tells agent i that she believes ψ. It provides some j’s belief to i, so that i can conduct simulation for j.

The operator $[tells(j, i, \psi)]$ is a normal modal box. Its semantic counterpart is a binary relation $R_{ji\psi}$ where $R_{ji\psi}st$ means that at state s, t is a next state after j tells i that (she believes) ψ. The formula $[tells(j, i, \psi)]\varphi$ is true at s if φ is true at every t such that $R_{ji\psi}st$.

We have three types of belief modalities: B_i, B_i^{Pu} and B_i^j. Correspondingly, there are three types of epistemic accessibility relations in the model: R_i, R_i^{Pu} and R_i^j. Belief formulas are evaluated in the standard way. A formula in the form $B_i\varphi$ ($B_i^{Pu}\varphi$ or $B_i^j\varphi$) is true at state s if φ is true at all s' such that R_iss' ($R_i^{Pu}ss'$ or R_i^jss'). Relations R_i, R_i^{Pu} and R_i^j represent different types of belief status of agent i.

B_i is the usual belief operator that is read as “agent i believes that …”. R_iss' means that i sees s' as being possible at state s.

B_i^j is a simulative belief operator. The formula of the form $B_i^j\varphi$ is read as, i can simulate that φ is a belief of j, or in other words, φ is a simulative belief of i for j. As shown in the example, the fact that φ is a simulative belief of i for j ($B_i^j\varphi$) does not necessarily mean that φ is an actual belief of i or an actual belief of j. Formally, both $B_i^j\varphi \rightarrow B_i\varphi$ and $B_i^j\varphi \rightarrow B_j\varphi$ should not be valid.

As for its semantic counterpart, R_i^jss' means that at state s, i thinks that j sees s' as possible. R_i^j represents the belief status held by i with respect to j. Note that, even though R_i^j is regarding j, it represents a type of i’s belief status. The calculation of $B_i^j\varphi$’s value is within i’s belief status.

In the real world, people usually do not use all their beliefs when simulating others; otherwise, everyone would think that everyone else believes in all what they believe. Here, we distinguish beliefs that are used in simulating others, called *public* beliefs, from those that are not, called *private* beliefs. For example, Alice may assume that Bob has the same basic beliefs about the physical world as she does. She will use her belief “the earth is bigger than the moon” in simulating Bob. On the other hand, Alice may not use her (private) belief that “John is interested in logic” in the simulation, because she does not assume that the belief is publicly believed by all agents. We introduce the public belief operator B_i^{Pu} for this purpose, where $B_i^{Pu}\varphi$ means that φ is a public belief of i, that is, i thinks that φ is a belief of everyone.