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A QBF-based formalization of abstract argumentation semantics

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ABSTRACT

We introduce a unified logical theory, based on signed theories and Quantified Boolean Formulas (QBFs) that can serve as the basis for representing and computing various argumentation-based decision problems. It is shown that within our framework we are able to model, in a simple and modular way, a wide range of semantics for abstract argumentation theory. This includes complete, grounded, preferred, stable, semi-stable, stage, ideal and eager semantics. Furthermore, our approach is purely logical, making for instance decision problems like skeptical and credulous acceptance of arguments simply a matter of entailment and satisfiability checking. The latter may be verified by off-the-shelf QBF-solvers.

1. Introduction

Dung's abstract argumentation theory [37] has been shown to be able to model a range of formalisms for nonmonotonic reasoning, including Default Logic [66], Pollock's OSCAR system [62,63], logic programming under stable model semantics [50,51], three-valued stable model semantics [75], and well-founded model semantics [69], Nute's Defeasible Logic [53], and so on. A key concept in Dung's theory is that of an *argumentation framework*, which is essentially a directed graph in which the nodes represent arguments and the arrows represent an attack relation between the arguments. When applied to model nonmonotonic reasoning, an argument can be seen as a defeasible proof for a particular claim. The precise contents of the argument depend on the particular logical formalism one is modeling. When applying argumentation to model logic programming, one can have arguments that consist of a number of logic programming rules (like a tree of rules, as in [75] or a list of rules, as in [65]). When applying argumentation to model default logic, one can have arguments that consist of a number of defaults (like a list of defaults, as in [2,33]). The attack relation (the arrows in the graph) then states which of these defeasible proofs can be seen as reasons against other defeasible proofs.

When applied in the context of nonmonotonic reasoning, argumentation can be seen as a three steps process. In the first step, one starts with a knowledge base (like a logic program or a default theory) and constructs the associated argumentation framework. In the second step one selects zero or more sets of arguments, according to a pre-defined criterion called an *argumentation semantics*. A key feature of an argumentation semantics is that it is defined purely on the structure of the graph (argumentation framework) without looking on the actual contents of the arguments. In the third step, one starts with the (zero or more) sets of arguments yielded by the argumentation semantics, and for each of these sets of arguments

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one constructs the associated set of conclusions. This is usually done by identifying for each argument (defeasible proof) in the set the claim that it aims to prove.

The three step procedure sketched above can be used to model a wide range of formalisms for nonmonotonic reasoning. As an example, if one starts from a knowledge base consisting of a logic program, and constructs arguments as sequences of rules that attack each other on their weakly negated statements (Step 1), then applies the principle of *stable semantics* on the resulting argumentation framework (Step 2) and takes for each selected argument the head of its top-rule (Step 3), the resulting sets of conclusions are precisely the stable models (in the sense of [50]) of the logic program one started with [37]. Similar results have been obtained for default logic [37], logic programming under well-founded model and three-valued stable model semantics [37,75], and Nute's defeasible logic [53].

One of the key advantages of the argumentation approach to nonmonotonic reasoning is that of modularization. The entailment process is remodeled in the form of three modular steps. Furthermore, the nonmonotonicity is isolated in the second step (applying argumentation semantics). The first step is monotonic, since having more information in the knowledge base leads to a superset of arguments and the associated widening of the attack relation. Similarly, the third step is monotonic, since a superset of arguments will yield an associated superset of conclusions. Only the second step is nonmonotonic, since the presence of additional arguments can cause other arguments not to be selected anymore by the argumentation semantics. Thus, the second step is the one that makes the overall process nonmonotonic.

Another advantage of the argumentation approach is that it becomes possible to specify nonmonotonic entailment in terms of dialogue (as is for instance done in [24,29,31] or other dialectical proof procedures like those in [34,35,38,56,65,61,71]). In contrast to traditional logical approaches, argumentation derives not so much what is *true* in a model theoretical way, but what is *defensible* in rational discussion. It turns out that some of the argumentation semantics that have been stated in the literature correspond to different ideas about what constitutes rational discussion.

Of the three step procedure, as pioneered by Dung [37], the second step has received the most subsequent research attention. Although one may say that an argumentation framework and the associated argumentation semantics (Step 2) should be seen as an *abstraction* of an argumentation formalism rather than a full argumentation formalism itself, such an abstraction can nevertheless be regarded as one of the simplest ways to examine the concept of nonmonotonicity, without having to deal with traditional notions of logical entailment. One particular issue to be aware of, however, is that as mentioned before, the argumentation semantics is defined purely on the structure of the graph, and does not examine the actual contents of the arguments. Although under some circumstances this can lead to the selection of sets of arguments with inconsistent conclusions (as is for instance pointed out in [32]) it has also been shown that for a wide range of semantics (more specifically: for semantics that are *admissibility-based*) the resulting conclusions will not only be consistent but also satisfy other desirable properties, provided that the argumentation framework is constructed (Step 1) according to particular principles (see [26,52,64] for more details). This makes admissibility-based semantics of particular interest compared to non-admissibility-based semantics.

Several admissibility-based semantics have been stated in the literature, including grounded, complete, preferred and stable semantics [37], semi-stable semantics [21,70], ideal semantics [38] and eager semantics [22]. One particular issue that has been studied recently is how these semantics can be expressed in a purely logical way. It was shown that complete and stable semantics can be expressed in propositional logic [14,27] and grounded, preferred, and semi-stable semantics can be expressed using second-order modal logic [54,55].

In this paper we provide a uniform and simple approach, based on signed theories and quantified Boolean formulas (QBFs), that is able to adequately capture *all* of the above mentioned argumentation semantics. QBFs are formulas involving only propositional languages and quantifications over propositional variables. Their application is vast, covering many areas among which are planning [67], verification [12,59], and different computational paradigms for nonmonotonic reasoning, such as default reasoning [15], circumscribing inconsistent theories [16] and computations of belief revision operators [36]. In our case, the use of signed theories and QBFs implies that decision problems like skeptical and credulous acceptance of arguments are a matter of logical entailment and satisfiability, which can be verified by existing QBF-solvers.

The rest of this paper is organized as follows: In the next section we review the main notions for our framework. We recall the two most common methods of giving a semantics to abstract argumentation frameworks (Sections 2.1 and 2.2), and review the means for expressing them by propositional logical theories, namely by signed formulas in the context of three-valued semantics (Section 2.3). Then, in Section 3, we show how complete semantics, which serves as the basis of many other admissibility-based semantics, can be described using three-valued semantics and signed theories. This also yields a simple way of representing stable semantics (Section 3.2). Based on these results, in Section 4 we continue to model grounded, preferred, semi-stable, ideal and eager semantics, using an approach based on quantified Boolean formulas, similar to the one taken in [3,7] for reasoning with paraconsistent preferential entailments, and in [8] for repairing inconsistent databases. To illustrate that our approach is not restricted to admissibility-based semantics, we also show how the notion of stage semantics [23,70] can be represented in our framework.

A clear advantage of approaches based on pure logic, including the present one, is that these allow one to reuse standard and well-studied notions, notations, techniques and results from formal logic, and apply them in the context of argumentation theory. In the last part of this paper (Section 5 onwards) we discuss some of the benefits of our approach and compare